Input-Output Linkages in Pigouvian Industrial Fluctuations

Can Tian*

University of North Carolina at Chapel Hill

September 2020

Abstract

Input-output linkages facilitate expectations-driven fluctuations. Signals about yet-to-be-realized aggregate (macro) and idiosyncratic (micro) fundamentals in the future affect current equilibrium outcome through chains of input needs when inputs require time to build. Depending on their importance on these chains, firms respond differently to common signals. The network structure, together with input time to build, determines the magnitude of signal-induced aggregate fluctuations. Macro signals need large input shares on average to produce sizable aggregate volatility, whereas micro signals require cross-sectional variations of input importance. Changing input time to build and altering the network structure have similar effects on equilibrium outcomes. (*Keyword: production network, input-output linkages, macro and micro shocks, expectations, aggregate volatility. JEL: D57, D83, D84, E32*)

*Email: tiancan@email.unc.edu Gardner Hall CB 3305, Department of Economics, University of North Carolina, Chapel Hill, NC 27599. Tel: 919-966-2383. This paper is based on Chapter 1 of my doctoral dissertation at the University of Pennsylvania. This paper was previously circulated under the title of "Forecast Shocks in Production Networks" (First draft: November 2013.) I am deeply indebted to Dirk Krueger for insightful advice and constant encouragement. I would also like to express my gratitude to Ufuk Akcigit, Harold Cole, Hanming Fang, Iourii Manovskii, and Guillermo Ordonez for valuable comments in the paper's early stages. I am thankful for the inspirational comments and discussions with Gary Biglaiser, Jie April Cai, Mario Crucini, Andrea Eisfeldt, Simon Gilchrist, Lutz Hendricks, Ricardo Lagos, Fei Li, Jim Nason, Peter Norman, Valerie Ramey, and Randy Verbrugge, and I also thank participants of numerous seminars and conference sessions. I give special thanks to the editor and two anonymous referees for their suggestions to improve the paper. Any remaining errors are mine.

1 Introduction

Expectation changes are an important driving force of aggregate fluctuations. The idea is traced back to Pigou (1927). A vast literature studying Pigou cycles emphasizes the expansionary impact of good news, whereas another aspect of Pigou's insights on the role of "links between business men" is less explored.[1] Input-output linkages among industries form an interconnected production network. These linkages are each industry's defining features; they connect the decisions of firms in one industry with those of their supplier and customer firms in other industries. Without modeling the interconnections explicitly, a realistic channel is missing for the impacts of expectation shifts to propagate among firms. This paper shows that information about yet-to-be-realized shocks of both macro- and microeconomic origins can have immediate impacts on the aggregate economy because of input-output linkages. These interconnections determine the scale and transition dynamics of such impacts.

The model incorporates public signals into a standard multi-sector real business cycle framework, with time-varying preferences of households as the payoff-relevant fundamentals. It relies on three main ingredients. First, exogenously given production technologies determine how firms are interconnected by input needs from one another.[2] Second, a time to build constraint on inputs requires firms to purchase some or all inputs in advance, relying on beliefs about yet-to-be-realized fundamentals. Third, a public set of noisy signals about future macro and micro fundamentals shapes the beliefs of all agents.

The key to the mechanism is the chains of forecasts resulting from chains of input needs and time to build. Time to build under uncertainty is the foundation of business-cycle dynamics in modern macroeconomics, pioneered by Kydland and Prescott (1982) and Long and Plosser (1983). As a consequence, information about the future is relevant for current decisions, and these decisions then affect future outcomes. With input-output linkages, a firm purchases inputs from other firms ahead of production. These decisions rely on the firm's beliefs about the demand for its product next period; this future demand depends on the households' preferences, as well as the input needs by downstream firms for their production in the future. The chain goes on. Therefore,

[1] The notion of Pigouvian fluctuations studied here is as follows. "The fact that many goods are made in response to forecasts of demand opens the way for errors of forecast. . . . [A]n error of optimism on the part of one group of business men itself creates a justification for some improved expectation on the part of other groups. For the group primarily affected . . . offers a higher real demand for the goods of other groups. . . . This means that large net errors of forecast are liable to occur much more frequently than they would do if these links [between business men] were lacking: and large net errors may be expected to carry with them large fluctuations in the aggregate volume of industrial activity." (Pigou, 1927, Part I. Chapter VI.)

[2] In what follows, "firm" and "industry" will be used interchangeably unless explicitly stated.

if two firms are connected by an input buyer-seller relation or a chain of input needs, then they are also connected over time. Signals containing information about an eventual buyer in a distant future can thus affect the initial seller's present sales, to an extent determined by the importance of the buyer. It is then intuitive that the structure of the entire production network determines whether and how the aggregate economy responds to the arrival of signals. At the rational expectations equilibrium, a closed-form expression summarizes this mechanism.

Signal-induced aggregate volatility is bounded from below and away from zero. The role of the production-network structure in determining the volatility lower bound is captured by simple network statistics — the cross-sectional mean and variance of *weighted indegrees*. Indegrees measure the importance of firms as input buyers, defined using the input-output matrix of an economy. The (i, j)-th element of this matrix is the input share of j's product used in i's production. The sum of each row i is the total input share for industry i, which is also its (weighted) *first order indegree*. Because of time to build, this matrix captures the strengths of inter-industry connections over one period. If a signal is informative about i in the next period, then i's first order indegree reflects the impact of this signal now due to i's input needs now. Therefore, the average degree corresponds to the impact of macro signals as macro shocks are common for all industries; the indegree dispersion relates to the impact of micro signals as micro shocks differ across industries. By the same logic, the strengths of inter-industry connections over T periods are summarized by the same input-output matrix to the T-th power, whose row sums analogously define indegrees of order T. An industry i's T-th order indegree captures the current impact of a signal about T periods in the future due to the chain of input needs of industries that are T steps upstream from i. The same intuition about the average and dispersion holds for higher-order indegrees. To summarize, given signals about the macro and micro fundamentals T periods in the future, the *mean* of T-th order indegrees sets the aggregate impact of the *macro* signal, whereas the *variance* sets that of the *micro* signal. After the arrival of signals, the beliefs of agents about the future are altered, and so these signals continue to affect firm decisions until those future fundamentals are realized. The mean and variance of indegrees of orders $\tau < T$ matter for impacts in the subsequent periods through the same mechanism.

A few concerns need to be addressed. The first one regards the impact of *micro* signals and aggregation level. When data become more disaggregated and industries become more narrowly-defined, the concern is that the impact of micro signals may diminish because industries average each other out. This would be the case if the variance in indegrees of some given order decreases

in the number of industries. However, in the case of the U.S. economy, the variance of indegrees of the first few orders does not decrease as industry-level data become more disaggregated; the dispersion actually appears to be increasing.

The second concern is that, if macro shocks, signals, and the resulting aggregate dynamics are of primary interest, then the details of the input-output network may not be important. To this end, I provide an irrelevance condition, under which the explicit input-output matrix disappears from the equilibrium characterization. But this condition essentially requires the final uses of all industries' products by the households to be "isomorphic" to their input uses. A realistic network structure does not satisfy this condition.

The last concern is about time to build. The entire mechanism relies on the time lag in input use, and so the results may change substantially if this assumption is relaxed. While time to build is crucial, that all inputs require the same time to build is not. A subset of such time-to-build inputs is sufficient to establish the inter-industry connections over time and make signals about the future relevant. The other inputs can be purchased just in time for production, effectively amplifying these over-time connections by creating an instantaneous cascade effect: a signal's impact on one firm's decisions immediately transmits to some of its input suppliers and further upstream suppliers without any delay. Changing the time-to-build assumption affects the strengths of intertemporal linkages among firms, as if by altering the network structure. The same statistics of the artificially altered network set the volatility lower bounds in response to macro and micro signals.

Fundamentals and signals in this paper have an ultimate demand-side origin as they regard household preferences. The choice to focus on shocks from the demand side rather than the supply side is purely for technical convenience. This is with little loss of generality because, with input-output linkages and time to build, signals about yet-to-happen events always affect current outcomes through chains of *input demand* regardless of their origins. For example, in an enriched model environment (e.g., with market imperfections or non-unitary elasticity of input substitution), signals about future aggregate or idiosyncratic productivities would alter some firms' expected input needs one period before, which changes their upstream firms' demand for inputs another period earlier, and so on. Therefore, with input-output linkages and time to build, signals about yet-to-be-realized shocks always have a demand-side origin, from the viewpoint of input suppliers.

This paper is closely related to the literature on expectations-driven business cycles. Beaudry

and Portier (2004, 2006) coin the term *Pigou cycles*, emphasizing the synchronized movements of the aggregate variables in response to news about future productivity. A few prominent examples with similar emphasis include Jaimovich and Rebelo (2009), Den Haan and Kaltenbrunner (2009), and Christiano et al. (2014), etc. Empirical evidence is also abundant, such as Barsky and Sims (2011, 2012) among many others. A common theme in the theoretical literature is to remove the contractionary effect of good news to generate optimism-induced booms (see Beaudry and Portier, 2007, 2014), often in a representative-producer environment. In contrast, this paper focuses on producer heterogeneity and particularly the role of input-output linkages in signal aggregation and impact propagation, thereby giving expectations-driven fluctuations a more disaggregated origin. Reproducing the aforementioned Pigou cycles is not the intention of this paper.

The network-based analysis of this paper connects to recent researches about transmission of shocks in along input-output linkages. The role of input-output linkages in shock transmission and prolongation began with Long and Plosser (1983). Earlier studies include Horvath (1998, 2000), Dupor (1999), Shea (2002). The literature is growing rapidly in recent years. A few examples are Carvalho (2010), Gabaix (2011), Foerster et al. (2011), Acemoglu et al. (2012), Carvalho and Gabaix (2013), Bartelme and Gorodnichenko (2015), Acemoglu et al. (2016), Atalay (2017), Baqaee (2018), and Baqaee and Farhi (2019). In these studies, production networks serve as a transmission, amplification, or prolongation mechanism for *realized* fundamental shocks. This paper's contribution is the analysis of the network structure in aggregating information about yet-to-happen events. Carvalho (2010) and Acemoglu et al. (2012) show that idiosyncratic shocks generate sizable aggregate volatility with a sufficiently asymmetric distribution of *outdegrees*. This paper highlights moments of *indegrees*, and micro signals generate more aggregate fluctuations when indegrees are more dispersed.

The rest of the paper is organized as follows. Section 2 sets up the model environment. Section 3 characterizes the equilibrium outcome. Section 4 discusses the role of the network structure and time to build. Section 5 explores the quantitative properties of the model. Section 6 concludes. Details of proofs, model extensions, and more are in the supplementary materials.

2 The Model

The model builds on the standard neoclassical multi-sector framework *à la* Long and Plosser (1983), with time to build of inputs. The model has demand-side uncertainty, similar to Shea

(2002), together with the arrival of signals. Time is discrete and infinite. The economy is populated by a unit-mass continuum of identical households and $n \in \mathbb{N}$ competitive industries, indexed by $i \in \{1, 2, \ldots, n\}$. Each industry i has a unit-mass continuum of identical firms and produces a distinct type of good i that can be used as other industries' input or put to final use by the households.

Technology. In each period, firms produce using inputs previously purchased from other firms and labor. Production technologies have non-increasing returns to scale, leaving room for other input factors not explicitly modeled, such as capital. I assume Cobb-Douglas production functions for tractability. Industry i's output $y_{i,t}$ to be traded in period t is given as

$$y_{i,t} = z_{i,t} h_{i,t}^{\gamma_i} \left(\prod_{j=1}^{n} x_{ij,t-1}^{\omega_{ij}} \right)^{\alpha_i}, \quad \text{with } \gamma_i, \alpha_i, \omega_{ij} \geq 0, \forall j, \text{ and } \gamma_i + \alpha_i \leq 1, \sum_{j=1}^{n} \omega_{ij} = 1.$$

Here, $z_{i,t}$ is the Hicks-neutral productivity, $h_{i,t}$ is labor hired just in time for production, with share γ_i, and $x_{ij,t-1}$ is the quantity of good j that industry i purchased as input in the previous period $t-1$. The total input share is α_i, and the share of a specific good j among all of i's inputs is ω_{ij}. A directed input-output link (edge) from industry j to industry i exists iff $\alpha_i \omega_{ij} > 0$. The baseline model assumes a common one-period time to build of all inputs.[3] An interpretation is that input delivery takes one period after orders are placed, and production is physically possible only after all necessary inputs are in place. Given the market price $p_{j,t}$ of every good j and the market wage rate w_t, firm i's flow profit in period t is its sales revenue net the cost of labor and its expenditure on inputs for period-$(t+1)$ production, given as $\pi_{i,t} = p_{i,t} y_{i,t} - w_t h_{i,t} - \sum_{j=1}^{n} p_{j,t} x_{ij,t}, \forall\, t$.

Households. Households are the final users of all goods and supply labor competitively. The discount factor is $\beta \in (0, 1)$. A flow-utility function summarizes the preferences of a representative household,

$$A_t \sum_{i=1}^{n} \theta_{i,t} \ln c_{i,t} - H_t, \quad \text{with } \theta_{i,t} \geq 0, \ \forall i, \text{ and } \sum_{i=1}^{n} \theta_{i,t} = 1,$$

where $c_{i,t}$ is the final use of good i and H_t is the total labor supply in period t.[4] $A_t > 0$ is a macro fundamental; it follows a stochastic process and affects aggregate final demand and labor supply.

[3] The one-period lag has alternative interpretations such as delayed input delivery. The mechanism requires *some* time discrepancy between input purchases and output sales. Section 4.4 shows that a subset of inputs requiring time to build is sufficient in this model. An alternative to time to build of inputs is "time to produce" of outputs, as originally assumed by Long and Plosser (1983), in which case labor is also hired one period in advance of production. This would further enhance the mechanism.

[4] Linear labor disutility reflects infinite Frisch elasticity of labor supply or indivisible labor by Hansen (1985) among others. It highlights the direct buyer-seller connections among firms by removing the indirect interactions via competition for labor. Model analysis is similar if labor disutility is $-\chi_0 H_t^{1+\chi_1}$, $\chi_0, \chi_1 > 0$.

The micro fundamentals are stochastic $\theta_{i,t}$, $\forall i$, $\forall t$; they determine product-specific preferences and $\theta_{i,t}$ will become the household's optimal expenditure share on good i in period t.[5] That micro fundamentals sum up to 1 means no love of variety from the household, and that shifts of the micro fundamentals are purely micro as they never result in a direct change in the aggregate final demand or labor supply. The household owns all firms. The usual budget constraints reduce to a standard income-expenditure identity in equilibrium such that $\sum_{i=1}^{n} p_{i,t}c_{i,t} = w_t H_t + \sum_{i=1}^{n} \pi_{i,t}$, $\forall\, t$.

Notation. Vectors are $n \times 1$ columns in bold lowercase letters; matrices are $n \times n$ squares in bold capitals. A vector of ones is $\mathbf{1}$. The identity matrix is $\boldsymbol{I}$. Expression $\boldsymbol{x} \equiv (x_i)$ defines a vector $\boldsymbol{x}$ using a typical element x_i, and $\boldsymbol{X} \equiv [x_{ij}]$ defines a matrix. Transposes of $\boldsymbol{x}$ and $\boldsymbol{X}$ are written as $\boldsymbol{x}'$ and $\boldsymbol{X}'$. Matrix inequalities indicate element-wise comparison.

Production technologies of all n industries are summarized by a list of vectors and a matrix: time-varying productivities $\boldsymbol{z}_t \equiv (z_{i,t})$, labor input shares $\boldsymbol{\gamma} \equiv (\gamma_i)$, and good input shares $\boldsymbol{\alpha} \equiv (\alpha_i)$ and $\boldsymbol{\Omega} \equiv [\omega_{ij}]$. Each row of $\boldsymbol{\Omega}$ sums up to 1. Combined, $\text{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega} = [\alpha_i \omega_{ij}]$ is the conventional *input-output matrix*, where $\text{diag}(\cdot)$ is the operator that converts a vector into a diagonal square matrix. The matrix $\text{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}$ fully characterizes the structure and weights of inter-industry connections required by production technologies. Household preferences at t are summarized by random variables $A_t > 0$ and $\boldsymbol{\theta}_t = (\theta_{i,t}) \in \triangle^{n-1}$, where $\triangle^{n-1}$ denotes the $(n-1)$-simplex. Endogenous variables are similarly represented when convenient.

Timing and information structure. Nature determines the entire history of payoff-relevant fundamentals $\{A_t, \boldsymbol{\theta}_t, \boldsymbol{z}_t\}_{t=0}^{\infty}$ before period 0. Agents do not observe future fundamentals in advance. They share a common prior belief about these fundamentals. The unconditional expectations satisfy $\mathbb{E}[A_{t+1}/A_t] = 1$, $\mathbb{E}[\boldsymbol{\theta}_t] = \boldsymbol{\theta} \equiv (\theta_i) \in \triangle^{n-1}$, and $\mathbb{E}[\boldsymbol{z}_t] = \boldsymbol{z} \equiv (z_i) \in \mathbb{R}^n_{++}$, $\forall t$. Fundamental series have finite means and variances conditional on the realized history. In period t, all agents observe a common set of signals $\mathcal{M}_t$ about some future fundamentals. Henceforth $\mathcal{M}_t$ is referred to as the *message set*. More on $\mathcal{M}_t$ and the belief updating of agents are in section 5.1. I impose the following assumption to ensure a well-defined, closed-form equilibrium characterization.

Assumption 1. *The $\{A_t\}$ process is independent of the $\{\boldsymbol{\theta}_t\}$ process. In any period t and for any*

[5] The household represents *all* final users in reality, without discrimination. In a reduced form, changes in $\theta_{i,t}$ capture any latent forces that shift the distribution of final users' expenditures on each industry's product. One can think of investments and government spendings as such forces. The eventual goal of $\theta_{i,t}$ is not about final uses or demand composition per se, but to drive tractable changes in input needs over time. In the Supplementary Materials, B.3 shows a more conventional treatment of the government and its resemblance to the baseline model; C.3 and C.5 contain further discussions regarding investments.

t', $t'' > t$, agents always believe that $A_{t'}$ is independent of $\boldsymbol{\theta}_{t''}$, regardless of the realized history of signals $(\mathcal{M}_\tau)_{\tau \le t}$; and agents do not expect fast growth of $A_{t'}$ conditional on the history, such that $\mathbb{E}[A_{t'} / A_t \mid (A_\tau, \mathcal{M}_\tau)_{\tau \le t}] \le \overline{A}\beta^{-(t'-t)}$ for some $\overline{A} > 0$.

The timing within a period is as follows. Entering period t, firms carry from the previous period their inputs, $\{x_{ij,t-1}\}_j, \forall\, i$. All agents observe A_t, $\boldsymbol{\theta}_t$, and $\boldsymbol{z}_t$, and a message set $\mathcal{M}_t$. Agents update their beliefs and make relevant decisions as price takers. In particular, firms purchase inputs for period-$(t+1)$ production.

Solution concept. The *rational-expectations competitive equilibrium* is a set of prices, allocations, and common beliefs such that (i) beliefs are rational and, after observing the exogenous signals, agents update their beliefs according to Bayes' rule, (ii) given current and expected prices, households and firms optimize, and (iii) each price clears its corresponding market. In particular, period-t market for good i clears when $y_{i,t} = c_{i,t} + \sum_{j=1}^{n} x_{ji,t}$.

3 Equilibrium Characterization

This section illustrates the mechanism through which changes in expectations of yet-to-be-realized future fundamentals affect current equilibrium outcomes. Details are in A.1. A key variable is the vector of *Domar weights* $\boldsymbol{s}_t = (s_{i,t})$, which is the ratio of gross sales of each industry divided by the total final use, $s_{i,t} \equiv \frac{p_{i,t}y_{i,t}}{\sum_i p_{i,t}c_{i,t}}$. The analytical form of $\boldsymbol{s}_t$ highlights how a production network aggregates expectations in equilibrium.

At the competitive equilibrium, Domar weights satisfy

$$\boldsymbol{s}_t = \underbrace{\boldsymbol{\theta}_t}_{\text{households' final uses}} + \overbrace{\beta \boldsymbol{\Omega}' \text{diag}(\boldsymbol{\alpha}) \frac{1}{A_t} \mathbb{E}_t[A_{t+1}\boldsymbol{s}_{t+1}]}^{\text{downstream firms' input uses}}. \tag{1}$$

This equation results from the market-clearing condition and firms' Euler equations for inputs. Relevant information about the future affects current market outcomes via input purchases due to time to build. Intuitively, every industry i's gross sales ($s_{i,t}$) depend on the household's final-use purchases ($\theta_{i,t}$) and its downstream industries' input purchases. Each downstream industry j purchases input i based on the expected future sales of j ($s_{j,t+1}$) and the weight of this input in j's production ($\alpha_j \omega_{ji}$). The same intuition applies to industries further downstream in more distant futures.

Common beliefs of agents permit the application of the law of iterated expectations. I henceforth use subscript $t+\tau \mid t$ to denote posterior expectations $\mathbb{E}_t[\cdot]$ of $t+\tau$ variables. The distributional assumptions and the belief updating process are deferred to section 5.1.

Theorem 1. *(Expectation aggregation.) The equilibrium Domar weight vector is a discounted and network-weighted sum of expected future fundamentals, given by*

$$\boldsymbol{s}_t = \boldsymbol{\theta}_t + \frac{1}{A_t} \sum_{\tau=1}^{\infty} \left(\beta \boldsymbol{\Omega}' \text{diag}(\boldsymbol{\alpha})\right)^{\tau} A_{t+\tau|t} \boldsymbol{\theta}_{t+\tau|t}, \quad \forall t. \tag{2}$$

Equation (2) has an intuitive interpretation. All demand originates from the final-use demand by households. Production of any good goes to households eventually, either directly as itself, or indirectly after being transformed into some other goods. Any expected change in future final-use demand triggers a chain of earlier changes in input demands. If firm i sees more sales ($s_{i,t}$), it means that households want more of product i ($\theta_{i,t}$) or are expected to want more of its downstream or further downstream firms' products in the future ($\theta_{j,t+\tau}$). The latter increases the actual input demand for product i.

Each industry's equilibrium sales depend on the realized macro and micro fundamentals, as well as beliefs about future fundamentals. Signals affect these beliefs, which affect the expectations of fundamentals at various times in the future. Theorem 1 shows how the network aggregates these expectations. Two implications follow. First, signals about preference shifts in a distant future regarding primarily a subset of industries at that time can affect a larger set of industries now. Second, because each industry has a distinct position in the network, a common aggregate shock or signal has differentiated impacts on each industry.

Assumption 1 of independence is crucial for the closed-form expression in equation (2). The realized preference shifts and anticipated future shifts are separable. Apart from that, the equation holds for arbitrary processes $\{A_{t+\tau}\}$ and $\{\boldsymbol{\theta}_{t+\tau}\}$ and for any content of the common message set $\mathcal{M}_t$, as long as firms' problems remain well-defined.

Productivity shocks to current $\boldsymbol{z}_t$ or any future $\boldsymbol{z}_{t+\tau}$ have no effect on equilibrium $\boldsymbol{s}_t$ due to Cobb-Douglas production functions in a frictionless setting. Therefore, $\mathcal{M}_t$ is henceforth assumed to contain no signals about future productivities.

The rest of the equilibrium outcome follows $\boldsymbol{s}_t$. Employment by industry is given as $\boldsymbol{h}_t = A_t \,\text{diag}(\boldsymbol{\gamma}) \boldsymbol{s}_t$, and the aggregate equilibrium labor is $H_t = A_t \boldsymbol{\gamma}' \boldsymbol{s}_t$. Each industry's output is then pinned down. Industry i's purchased quantity of good j is given by $x_{ij,t} = y_{j,t} \frac{\beta \alpha_i \omega_{ij} \mathbb{E}_t[A_{t+1} s_{i,t+1}]}{A_t s_{j,t}}$. The

final use of good i by households satisfies $c_{i,t} = y_{i,t}\frac{\theta_{i,t}}{s_{i,t}}$.

Production functions evaluated at the equilibrium describe the dynamics of output levels,

$$\ln \boldsymbol{y}_t = \overbrace{\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega} \ln \boldsymbol{y}_{t-1} + \boldsymbol{e}_t}^{\text{input flows}} + \overbrace{\boldsymbol{\gamma} \ln A_t + \operatorname{diag}(\boldsymbol{\gamma}) \ln \boldsymbol{s}_t}^{\text{labor}} + \ln \boldsymbol{z}_t + \boldsymbol{\psi}, \tag{3}$$

where $\boldsymbol{y}_t \equiv (y_{i,t})$ is gross output by industry, $\boldsymbol{e}_t \equiv \operatorname{diag}(\boldsymbol{\alpha}) \ln\left(\mathbb{E}_{t-1}[A_t \boldsymbol{s}_t]\right) - \operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega} \ln(A_{t-1}\boldsymbol{s}_{t-1})$ captures previous allocations of inputs, and $\boldsymbol{\psi}$ is a constant vector set by production technologies. The dynamics of growth rates $\Delta \ln \boldsymbol{y}_t \equiv \ln \boldsymbol{y}_t - \ln \boldsymbol{y}_{t-1}$ follow immediately. Equation (3) illustrates the driving forces of real output fluctuations, and the transmission channel of *realized* disturbances across industries and over time. Expectations affect real output quantities through labor hiring and input purchases. Both are functions of period-t expectations, but labor $\boldsymbol{h}_t$ affects same-period outputs whereas purchased inputs affect outputs with a one-period delay. If productivity $\boldsymbol{z}_t$ is the only source of dynamics, the equation becomes the one studied by Long and Plosser (1983), or $\Delta \ln \boldsymbol{y}_t = \operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}\Delta \ln \boldsymbol{y}_{t-1} + \Delta \ln \boldsymbol{z}_t$. The static version is featured in, for example, Acemoglu et al. (2012). In these models, the current one included, input flows serve as a transmission mechanism of *realized* output changes. The next section shows that these linkages are also important for yet-to-be-realized fundamentals to have sizable impacts on current outcomes.

4 Network Structures and Expectation-Driven Aggregate Fluctuations

Input-output linkages govern firms' heterogeneous responses to signals about future fundamentals. This section examines the exact role of the network structure in determining the magnitude of aggregate fluctuations driven by expectation changes.

The aggregate variables to gauge the impact of signals are as follows. The aggregate labor is as before, $H_t = A_t \boldsymbol{\gamma}' \boldsymbol{s}_t$. Let the numeraire good be a fixed basket $\boldsymbol{\theta}$ of all goods using the Cobb-Douglas aggregator.[6] The aggregate final-use quantity C_t and the associated price index P_t become $C_t = \prod_{i=1}^n c_{i,t}^{\theta_i}$ and $P_t = \prod_{i=1}^n (p_{i,t}/\theta_{i,t})^{\theta_i}$. It can be verified that $P_t C_t = \sum_i p_{i,t} c_{i,t}$ in equilibrium. The assumption of in-advance input purchases creates a discrepancy between C_t and the measure of real GDP in this model.[7] I follow the standard practice to calculate current

[6] I use a fixed basket instead of the realized $\boldsymbol{\theta}_t$ to avoid the mechanical fluctuations in the aggregate final use caused by changes in basket composition. Results of this section do not rely on this choice.

[7] In the data, an industry's value added over a given period of time is the difference between the value of the gross output of this industry and the cost of inputs already used up to produce said output. Purchased but not yet used inputs are not considered part of the cost. See Karen J. Horowitz (2009).

market values of used-up inputs, and define industry i's real value added in period t as a quantity measure $q_{i,t} = p_{i,t}y_{i,t}/P_t - \sum_j p_{j,t}x_{ij,t-1}/P_t$. The real aggregate GDP becomes $Q_t = \sum_i q_{i,t} = C_t\left(\sum_i s_{i,t} - \sum_i \frac{s_{i,t}/s_{i,t-1}}{y_{i,t}/y_{i,t-1}}(s_{i,t-1} - \theta_{i,t-1})\right)$.

4.1 How Much Does the Network Structure Matter for the Impact of Signals?

The role of the network structure in determining the magnitude of signal-induced aggregate fluctuations can be distilled into simple statistics.[8] These statistics set the lower bounds of aggregate volatility in response to macro and micro signals. In particular, more industries do not necessarily average out impacts of micro signals. Details are in A.2.

To isolate the impact of signals, consider the following scenario. The economy is at its steady state, with preferences fixed by A and $\boldsymbol{\theta}$, and productivities at $\boldsymbol{z}$. All aggregate variables are at their respective steady-state values C, H, and Q. In period t, $\mathcal{M}_t$ arrives with signals about some future fundamentals A_{t+T} and $\boldsymbol{\theta}_{t+T}$, $T \in \mathbb{N}$. Henceforth T is called the *horizon* of a signal. Let $\widehat{c}_t$, $\widehat{h}_t$, and $\widehat{q}_t$ be the first order approximations of resulting changes in corresponding aggregate variables, such that, for example, $\widehat{c}_t \approx \ln \frac{C_t}{C}$. The measures of aggregate impacts of signals are the standard deviations $\sigma(\cdot)$ of these hat variables.

I make further restrictions to ease expressions and highlight the role of the network structure. All the n industries have the same labor input share $\gamma \in (0, 1 - \max_{1\leq i\leq n}\alpha_i]$. A_t has a finite mean $\mathbb{E}[A_t] = A > 0$ and variance $\text{var}(A_t) = (A\sigma_A)^2$, $\forall t$, for some $\sigma_A \geq 0$. Preference shifts are symmetrically distributed across n industries, such that for some $\sigma_\theta \geq 0$,

$$\mathbb{E}[\boldsymbol{\theta}_t] = \boldsymbol{\theta} = \frac{1}{n}\mathbf{1}, \quad \text{and} \quad \text{var}(\theta_{i,t}) = \frac{1}{n}\sigma_\theta^2, \ \forall i, \quad \text{corr}(\theta_{i,t}, \theta_{j,t}) = -\frac{1}{n-1}, \ \forall i \neq j.$$

Micro fundamentals fluctuate less when $n \to \infty$ as the volatility $\text{var}(\theta_{i,t})$ *decreases* in n.[9] Negative cross-industry correlation results from symmetry and $\text{var}(\mathbf{1}'\boldsymbol{\theta}_t) = 0$. The series $\{A_t\}$ and $\{\boldsymbol{\theta}_t\}$ can be serially independent or persistent.

An exact belief updating rule relies on additional assumptions on distributions, deferred to section 5.1. For now, it suffices to summarize the process by $r_A, r_\theta \in [0, 1]$ such that the ex ante

[8] I thank an anonymous referee for inspiring this exercise.

[9] This is consistent with the Dirichlet distribution of $\boldsymbol{\theta}_t$ assumed in section 5.1 for quantitative illustrations. If a Dirichlet distribution has a symmetric mean $\boldsymbol{\theta} = \frac{1}{n}\mathbf{1}$, then $\text{corr}(\theta_{i,t}, \theta_{j,t}) = -\frac{1}{n-1}$, $\forall\, i, j$, and $\text{var}(\theta_{i,t})$ goes to zero at rate $\frac{1}{n}$ as $n \to \infty$, with other parameters fixed. A.2 allows $\text{var}(\theta_{i,t})$ to change in n at an arbitrary rate $\frac{1}{n^\delta}$.

variances of the posterior expectations satisfy

$$\text{var}(A_{t+T|t}) = r_A(A\sigma_A)^2, \quad \text{and} \quad \text{var}(\theta_{i,t+T|t}) = r_\theta \frac{\sigma_\theta^2}{n}, \quad \forall i.$$

Intuitively, r_A and r_θ capture the relative precisions of signals and the speed of belief updating; higher values indicate more precise signals and faster belief updating.

The matrix $\text{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}$ describes the input-output linkages as a weighted directed graph. The *weighted* T-th order *indegrees* $\boldsymbol{d}^{\text{in},T}$ and *outdegrees* $\boldsymbol{d}^{\text{out},T}$ are defined as

$$\boldsymbol{d}^{\text{in},T} \equiv (\text{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega})^T \mathbf{1}, \quad \text{and} \quad \boldsymbol{d}^{\text{out},T} \equiv (\boldsymbol{\Omega}' \,\text{diag}(\boldsymbol{\alpha}))^T \mathbf{1}.$$

In particular, the first order indegree of industry i is its goods input share, $d_i^{\text{in},1} = \alpha_i$; the first order outdegree of i determines how much of product i is sold to other industries as inputs, $d_i^{\text{out},1} = \sum_j \alpha_j \omega_{ji}$. More generally, $d_i^{\text{in},T}$ summarizes all possible ways (directed walks) for any good to flow T steps downstream *to* industry i; $d_i^{\text{out},T}$ summarizes all possible ways for good i to flow T steps downstream *from* industry i. Define the discounted Leontief inverse as

$$\boldsymbol{L} = [\boldsymbol{I} - \beta \,\text{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}]^{-1}.$$

Then the steady-state Domar weight vector $\boldsymbol{s}$ becomes

$$\boldsymbol{s} = \boldsymbol{L}'\boldsymbol{\theta} = \frac{1}{n}\boldsymbol{d}^{\text{out, all}}, \quad \text{where } \boldsymbol{d}^{\text{out, all}} \equiv \mathbf{1} + \sum_{\tau=1}^{\infty} \beta^\tau \boldsymbol{d}^{\text{out},\tau}.$$

This equation connects $\boldsymbol{s}$ to each industry's *Katz-Bonacich centrality* with a discount factor β.[10]

For a given T, denote $\overline{d^T}$ as the *cross-sectional mean* of order-T weighted degree such that

$$\overline{d^T} \equiv \frac{1}{n}\sum_{i=1}^{n} d_i^{\text{in},T} = \frac{1}{n}\sum_{i=1}^{n} d_i^{\text{out},T},$$

and $V^{\text{in},T}$ as the *cross-sectional variance* of indegrees of order T such that

$$V^{\text{in},T} \equiv \frac{1}{n-1}\sum_{i=1}^{n} (d_i^{\text{in},T} - \overline{d^T})^2.$$

[10] Specifically, $\boldsymbol{d}^{\text{out, all}} - \mathbf{1}$ is the *Katz-Bonacich centrality* vector with a discount factor β. See, for example, Bloch et al. (2019) for the definition of this centrality measure and its uses in economics.

As weighted indegrees, $d_i^{\text{in},T} \leq 1-\gamma$ for any order T and industry i, which implies that $\overline{d^T} \leq 1-\gamma$ and $V^{\text{in},T} < \frac{n}{4(n-1)}$. Hence, $\overline{d^T}$ and $V^{\text{in},T}$ exist and are finite regardless of the size and structure of the network.

Theorem 2. *(Volatility Lower Bound.) Given the relative precisions of signals captured by r_A and r_θ, the magnitude of aggregate fluctuations in period t in response to horizon-T signals in $\mathcal{M}_t$ is determined by D_T, defined by*

$$(D_T)^2 \equiv (\beta^T)^2 \left(r_A \sigma_A^2 (\overline{d^T})^2 + r_\theta \sigma_\theta^2 V^{in,T} \right),$$

such that, ceteris paribus, D_T sets the volatility lower bound in aggregate final use $\widehat{c}_t$, aggregate labor $\widehat{h}_t$, and aggregate real GDP $\widehat{q}_t$,

$$\sigma(\widehat{c}_t) \geq \frac{1-\gamma}{d_{\max}^{out,\,all}} D_T, \qquad \sigma(\widehat{h}_t) \geq \frac{D_T}{\overline{d^{all}}}, \qquad \sigma(\widehat{q}_t) \geq \gamma D_T,$$

where $d_{\max}^{out,\,all} = \max_{1\leq i\leq n} d_i^{out,\,all}$ and $\overline{d^{all}} = \frac{1}{n}\sum_i d_i^{out,\,all}$.

Theorem 2 establishes that the cross-sectional *mean* and *variance* of indegrees $\boldsymbol{d}^{\text{in},T}$ summarize the role of the network structure in determining the impact scale of horizon-T macro and micro signals with given precisions. Input time to build transforms the physical connections among industries due to input flows into connections over time. Each link forged by input-output relation takes one period to complete. Therefore, horizon-T signals have an immediate impact only if such T-link connections into the future exist for some industries; the impact is larger if these connections are stronger. Intuitively, the cross-sectional average $\overline{d^T}$ measures the overall strength of all such connections, and it corresponds to *macro* signals because these signals go through all connections. Whether some connections are stronger than others does not matter for macro signals. In contrast, micro shocks contain differentiated information about different industries in the future, affecting each over-time connection differently. Heterogeneity in these connections captured by the variance $V^{\text{in},T}$ of *indegrees* is therefore more relevant for micro signals.

In the special case of $T = 1$, $\overline{d^1} = \overline{\alpha} \equiv \frac{1}{n}\sum_i \alpha_i$ is the mean input share, and $V^{\text{in},1}$ is the variance of input shares. If all industries have the same input share $\alpha_i = \alpha \in (0, 1-\gamma], \forall i$, then the first order indegrees have a positive mean α and zero variance. In this case, aggregate variables respond to macro but not micro signals about $t+1$ preferences.

If signals in the message set $\mathcal{M}_t$ have multiple horizons, then statistics of indegrees of all corresponding orders are relevant. For example, if $\mathcal{M}_t$ contains signals of all horizons $\tau = 1, \ldots, T$

with common precisions r_A and r_θ, then the volatility lower bounds in Theorem 2 hold with $(D_T)^2$ replaced by $\sum_{\tau=1}^{T}(D_\tau)^2$.

That degree heterogeneity is informative about aggregate volatility of micro origins is consistent with the theoretical analysis by Carvalho (2010) and Acemoglu et al. (2012), in a static setting. They conclude that the variation in *weighted outdegrees* is crucial for micro productivity shocks to drive aggregate fluctuations. Comparing the two sets of findings, the juxtaposition of in- and out-degree statistics mirrors that of *yet-to-be-realized* shocks and *realized* disturbances. In a *dynamic* environment, the distinction between preference shifts and productivity shocks as the source of fluctuations is less essential and more for technical convenience. With time to build, all yet-to-be-realized shocks can be viewed to have *demand-side* origins due to input needs before production; and similarly, all realized shocks have supply-side origins due to physical input flows from suppliers to customers.

The number of industries n does not directly affect the volatility lower bounds in Theorem 2. Different data aggregation levels cause n to change, and so do the births and deaths of industries and products. By assumption, a larger n lowers the volatility in micro fundamentals $\text{var}(\theta_{i,t}) = \frac{\sigma_\theta^2}{n}$. Yet the common volatility component D_T depends on n through network statistics $\overline{d^T}$ and $V^{\text{in},T}$ only. Therefore, unless these two statistics decrease in n, signal-induced aggregate fluctuations do not automatically become smaller as $n \to \infty$. If weighted indegrees of order T are random draws from a fixed "true" distribution over the domain $[0, 1-\gamma]$, then the mean $\overline{d^T}$ and variance $V^{\text{in},T}$ of a random sample $\left\{d_i^{\text{in},T}\right\}_{i=1}^{n}$ vary little when the sample size n is sufficiently large. Consequently, D_T and aggregate volatilities do not change much in n. Section 5.3 explores empirically the impact of n on the indegree distribution. There is no evidence for a declining mean or variance of indegrees as data become more disaggregated; they may in fact *increase* in n. A particular implication is that the impact of micro signals does not get averaged out automatically as $n \to \infty$.

After period t, signals received at t continue to affect equilibrium dynamics through two channels. Examples in section 4.2 illustrate the dynamics. The first channel is *physical* due to input flows and time to build. All realized impacts propagate *downstream* through the network. The law of motion of industrial output in equation (3) shows how each industry is affected through this channel. The physical transmission channel is analogous to that of *realized* productivity shocks, presented by Long and Plosser (1983). The analysis of *weighted outdegrees* by Carvalho (2010) and Acemoglu et al. (2012) thus applies. In the aggregate, $\boldsymbol{d}^{\text{out},1}$ captures the per-period rate of decay of realized disturbances through input flows.

The second channel is *belief-based*, resulting from Bayes' rule and unique to this model. This channel entwines with the physical one and remains open before period $t+T$ when the underlining fundamentals are realized. Weighted indegrees $\left\{\boldsymbol{d}^{\text{in},\tau}\right\}$ of *every* order $\tau = 1, \ldots, T$ contribute to the aggregate dynamics because of this channel. Specifically, when signal horizons are $T \geq 2$, period t posterior beliefs about $t+T$ fundamentals become the prior beliefs in period $t+1$, to be updated only if new information about $t+T$ arrives in $t+1$. Moreover, now that only $T-1$ periods are left till $t+T$, over-time connections of length $T-1$ become the most relevant, captured by the statistics of corresponding indegrees $\overline{d^{T-1}}$ and $V^{\text{in},T-1}$. Expectations about $t+T$ are also *less* discounted (β^{T-1}) in current sales $\boldsymbol{s}_{t+1}$. Therefore, for any $\tau = 1, \ldots, T-1$, posterior expectations $A_{t+T|t+\tau}$ and $\boldsymbol{\theta}_{t+T|t+\tau}$ are functions of period-t signals of horizon T, and so are Domar weights $\boldsymbol{s}_{t+\tau}$. Consequently, this belief-based channel affects both labor and input flows in the output dynamics in equation (3). This is an alternative mechanism for a transitory, one-time shock (signal) to have persistent impacts — a feature shared by dynamic models with information frictions and learning.

In period $t+T$, the underlining fundamentals in the signals are realized. Previous signals and thus expectations may turn out to be very wrong at this point, but the signals' real impacts have already shaped the realized history. From period $t+T$ onwards, the physical input-flow channel remains open and the aggregate impact decays monotonically.

4.2 Simple Examples of Network Structures

Theorem 2 shows that the cross-sectional mean and variance of indegrees capture the impact scale of macro and micro signals. Two implications follow. First, economies with the same network topology respond differently to the same signals if their input-output matrices differ quantitatively. Second, the aggregate responses of economies with distinct production technologies can be very similar if their input-output matrices generate similar degree statistics. In what follows, I use a set of production chains (Figure 1a) to demonstrate the first statement, and three symmetric networks with various inter-industry connections (Figure 1b, Figure 1c, and Figure 1d) to show the second. Details are in A.3.

The environment is as described in section 4.1. Both macro and micro signals are of horizon $T = 4$. They arrive at $t_0 = 0$ and alter the beliefs of agents. All other sources of fluctuation are shut down. To illustrate the impact of specific signals, it is without loss to specify the resulting posterior expectations $\boldsymbol{\theta}_{t_0+T|t_0}$ and $A_{t_0+T|t_0}$ directly. It is understood that the same change in the posterior

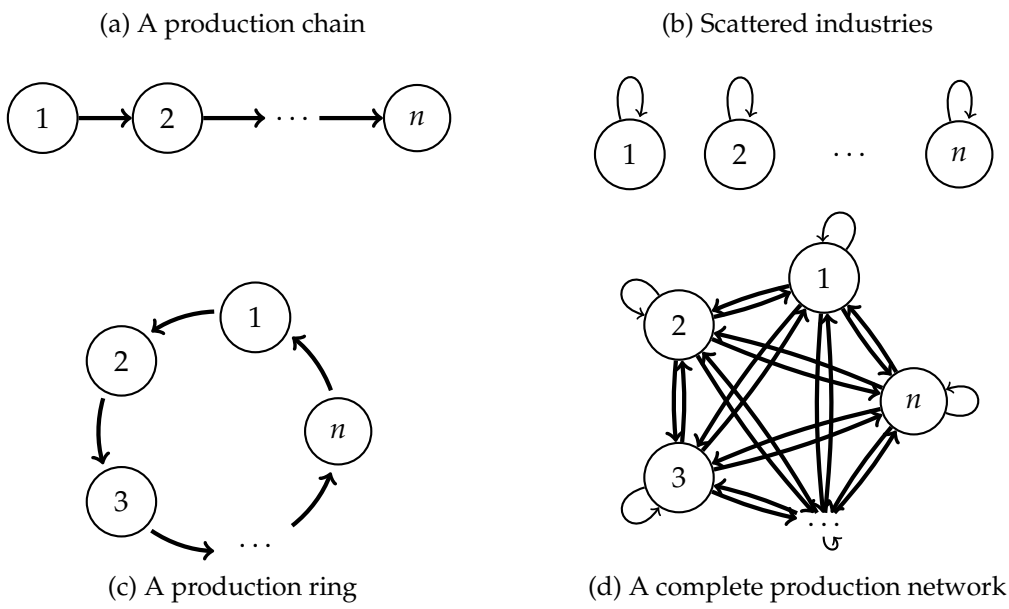
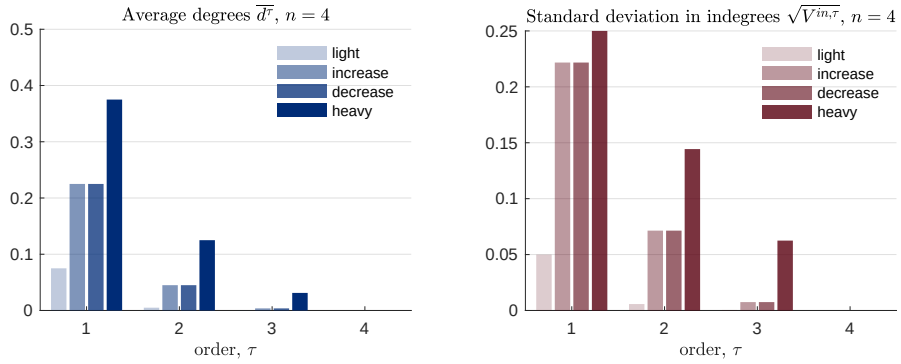


Figure 1: Examples of Network Structures.

Each circle represents an industry labeled by its index $i = 1, \ldots, n$. The direction of an arrow is the direction of input flow; an arrow from j to i means $\alpha_i \omega_{ij} > 0$. Figure 1a illustrates a production chain, where a well-defined upstream and downstream relationship exists for every pair of industries. Figure 1b represents independent industries with no inter-industry connections and a diagonal input-output matrix. Figure 1c plots input-output linkages that form a closed ring, where every industry has exactly one immediate customer and one immediate supplier. Figure 1d shows a complete network with the maximum number of inter-industry connections, and every element of its input-output matrix is positive. The associated input-output matrices and other parameter values are in Supplementary Materials A.3.

(a) Production chains of length $n = 4$



(b) Production chains of length $n = 10$

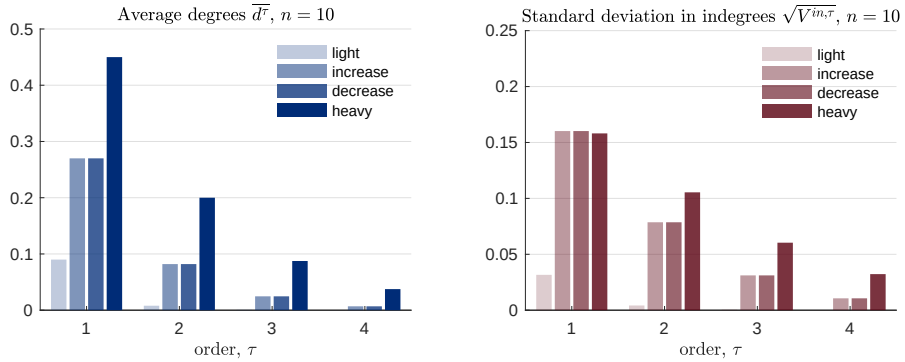


Figure 2: Statistics of weighted indegrees for various production chains.

Figure 2a compares the degree statistics $\overline{d^\tau}$ and $V^{\text{in},\tau}$ for four otherwise identical production chains when $n = 4$; Figure 2b shows the same for $n = 10$.

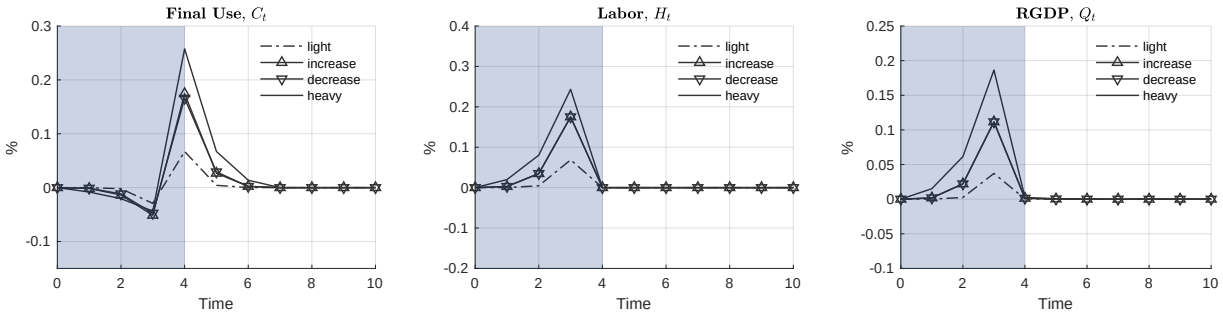
Each horizontal axis shows orders of degrees, and the bar height indicates values of corresponding statistics. Names and bar shades correspond to values of $\boldsymbol{\alpha}$ in each economy. A "light" chain means light input uses by downstream industries and "heavy" means heavy input uses, such that $\boldsymbol{\alpha}^{\text{light}} = \alpha^{\text{light}}\mathbf{1}$ and $\boldsymbol{\alpha}^{\text{heavy}} = \alpha^{\text{heavy}}\mathbf{1}$ for some $0 < \alpha^{\text{light}} < \alpha^{\text{heavy}} \leq 1 - \gamma$. The name "increase" describes a production chain with increasing input shares from upstream ($j = 1$) to downstream ($j = n$), such that $\alpha_j^{\text{increase}}$ increases in j; the name "decrease" means the opposite, with decreasing input shares from upstream to downstream such that $\alpha_j^{\text{decrease}}$ decreases in j. Details are in A.3.

expectation requires different signal realizations for different relative precisions of signals.

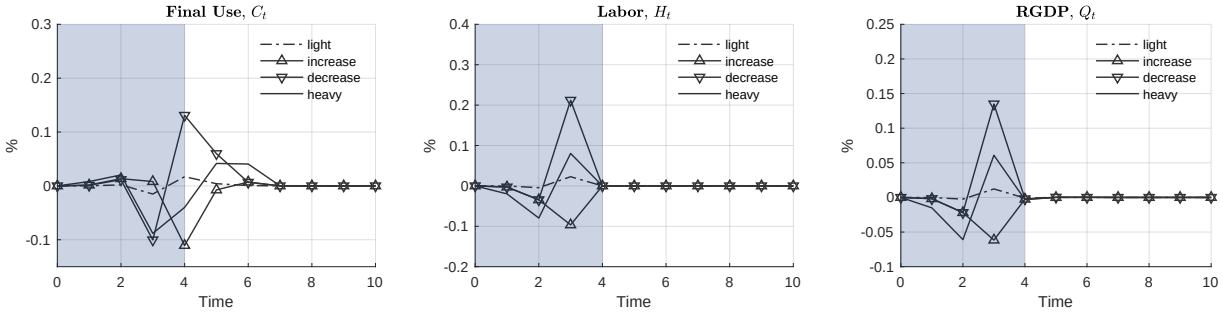
The aggregate responses to a macro signal and to a micro signal are separately examined in two "fake news" scenarios. In the macro-only scenario, a macro signal increases $A_{t_0+T|t_0}$. In the micro-only scenario, a micro signal shifts the final-use expenditure shares $\boldsymbol{\theta}_{t_0+T|t_0}$ and increases the expected share on one primarily affected industry i's product, crowding out the shares on others. The signals are "fake" because $A_{t_0+T} = A$ and $\boldsymbol{\theta}_{t_0+T} = \frac{1}{n}\mathbf{1}$.

A set of finite production chains (Figure 1a) illustrates an extreme case: identical network topology can produce very different dynamics. The chains have identical lengths and input-output connections, but different input shares $\boldsymbol{\alpha}$. Figure 2 shows the degree statistics for these

(a) Scenario 1: macro only, $n = 4$.



(b) Scenario 2: micro only, $n = 4$, primarily affected $i = 2$.



(c) Scenario 2: micro only, $n = 4$, primarily affected $i = 4$.

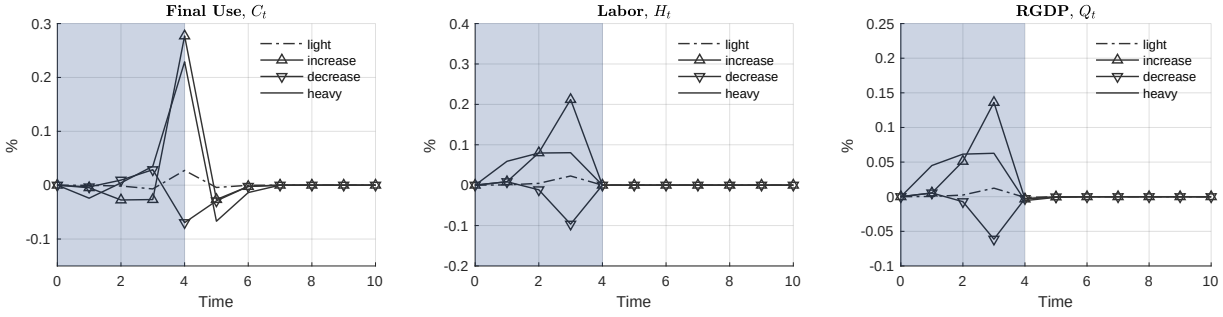
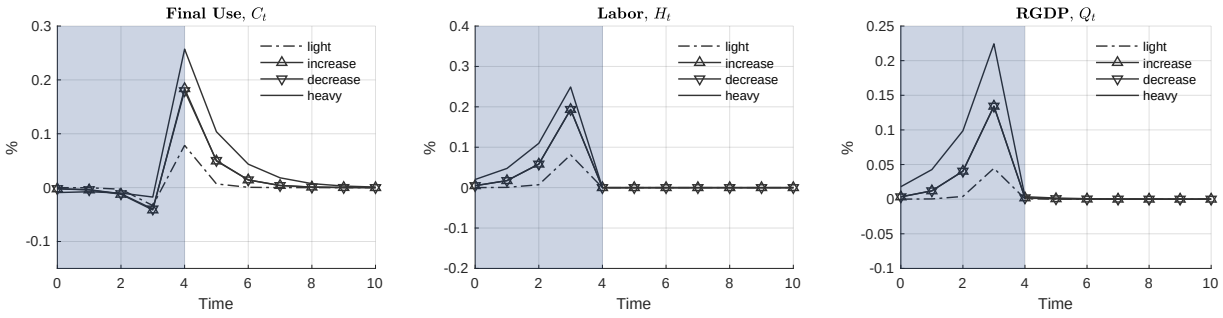


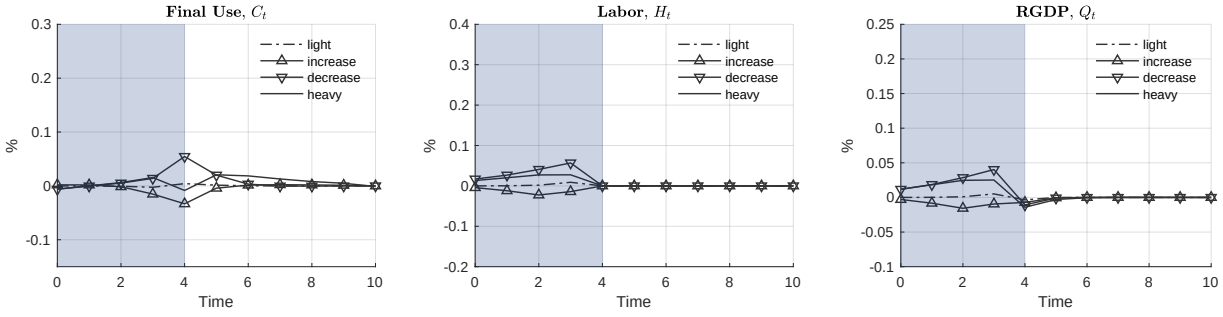
Figure 3: Aggregate responses of four production chains with identical length ($n = 4$) and topology but different values of $\boldsymbol{\alpha}$.

Every series is plotted as percentage deviation from the steady state calculated as log difference. Each horizontal axis represents time t. Shaded areas represent the periods from t_0 to $t_0 + T$. See the note of Figure 2 for explanations of the legend. Details are in A.3.

(a) Scenario 1: macro only, $n = 10$.



(b) Scenario 2: micro only, $n = 10$, primarily affected $i = 5$.



(c) Scenario 2: micro only, $n = 10$, primarily affected $i = 10$.

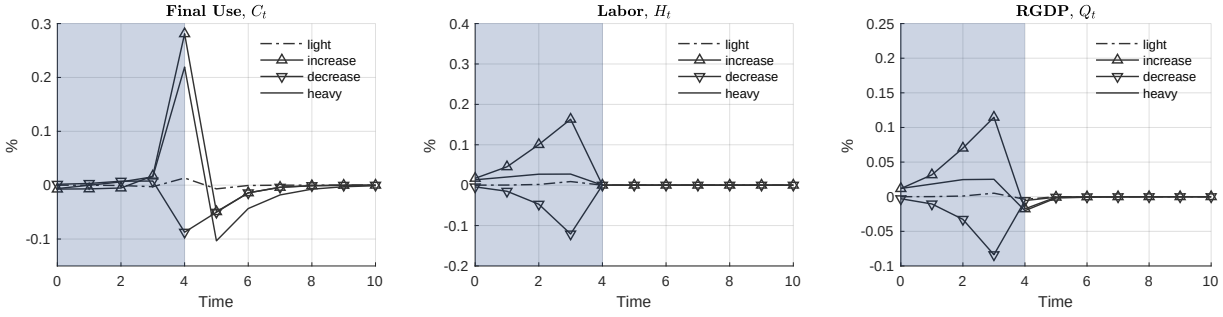


Figure 4: Aggregate responses of four production chains with identical length ($n = 10$) and topology but different values of $\boldsymbol{\alpha}$.

Every series is plotted as percentage deviation from the steady state calculated as log difference. Each horizontal axis represents time t. Shaded areas represent the periods from t_0 to $t_0 + T$. See the note of Figure 2 for explanations of the legend. Details are in A.3.

chains, named based on $\boldsymbol{\alpha}$ values. Figure 3 and Figure 4 show their responses in each "fake news" scenario. The quantitative differences among the degree statistics explain the differences in responses. The average degree $\overline{d^{T-\tau}}$ matters the most for the impact scale of macro signals in period $t_0+\tau$, $\tau=0,\ldots,T-1$, and all economies move in the same direction. In comparison, after the arrival of micro signals, the indegree variation $V^{\text{in},T-\tau}$ contributes to (i) the differentiated dynamics of aggregate variables across economies in response to the same signals, and (ii) the variation in the aggregate responses within the same economy when micro signals primarily concern different industries.

The second set of economies shows the other extreme: economies with drastically different interconnections can exhibit similar dynamics. Figure 5 shows the responses of three symmetric economies ranging from having no inter-industry connections to having all possible connections. They share an identical $\boldsymbol{\alpha}=\alpha\mathbf{1}$, for some $\alpha\in(0,1-\gamma]$. Therefore, due to symmetry, there is no dispersion in degrees of any order in each economy, with $\overline{d^{\tau}}=\alpha^{\tau}$ and $V^{\text{in},\tau}=0$, $\forall\tau\geq 1$. Both statistics are independent of n. Consequently, all economies respond identically to macro signals and very similarly to micro signals.[11]

4.3 When Is the Network Structure Irrelevant?

The second set of simple economies in the previous section suggests the possibility of network structure irrelevance, especially if *macro* shocks are the only source of fluctuations. This section complements Theorem 2 by providing a condition for network structure irrelevance, for the impact of *macro* fundamentals and signals. Only then can a representative-firm model describe an economy with input-output linkages. However, it is difficult for a realistic production network to meet this condition. Details are in A.4.

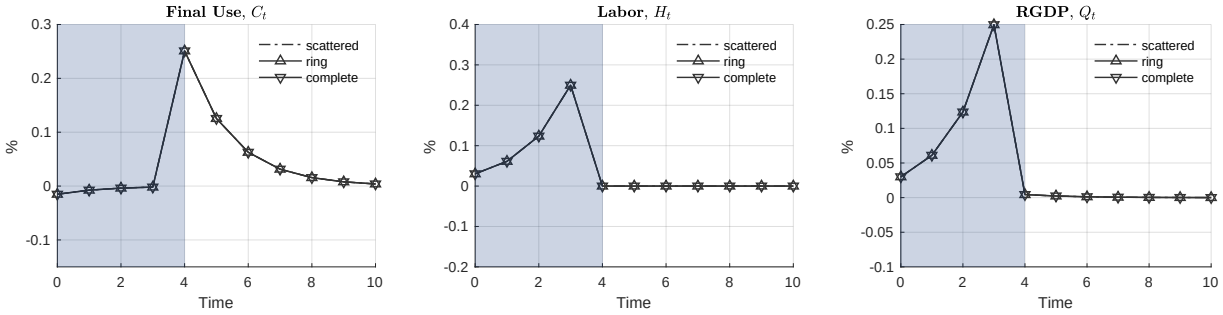
Consider a generic economy with fixed final-expenditure shares $\boldsymbol{\theta}$. The source of uncertainty and fluctuations includes the common productivity z_t, current preference A_t, and macro signals about future preference shifts.

Theorem 3. *(Condition for Network Structure Irrelevance.) Without micro shocks, the aggregate final use C_t does not directly depend on $\boldsymbol{\Omega}$ only if*

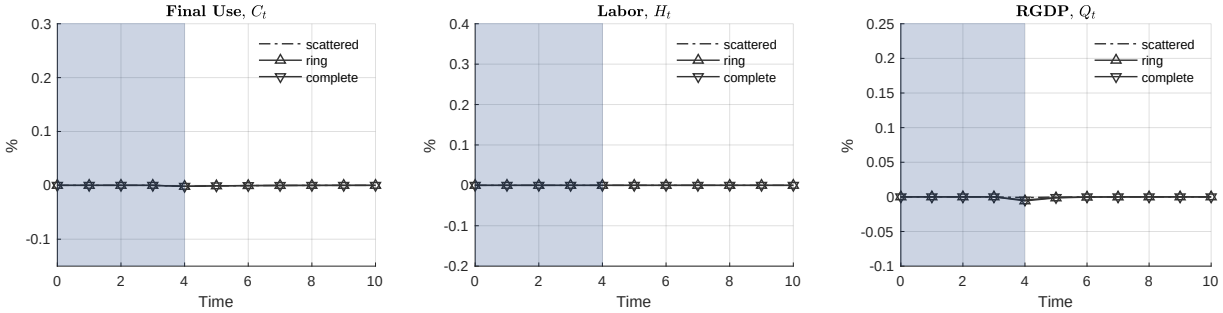
$$\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\theta}=\widetilde{\alpha}\boldsymbol{\theta},\quad \text{for some } \widetilde{\alpha}\in(0,\alpha_{\max}]\,, \tag{4}$$

[11] The slight differences among the responses of real GDP result from the input-price adjustment to accommodate time to build. It introduces additional nonlinearity that relies on cross-industry connections.

(a) Scenario 1: macro only, $\forall n$.



(b) Scenario 2: micro only, $n = 4$, $\forall$ primarily affected i.



(c) Scenario 2: micro only, $n = 10$, $\forall$ primarily affected i.

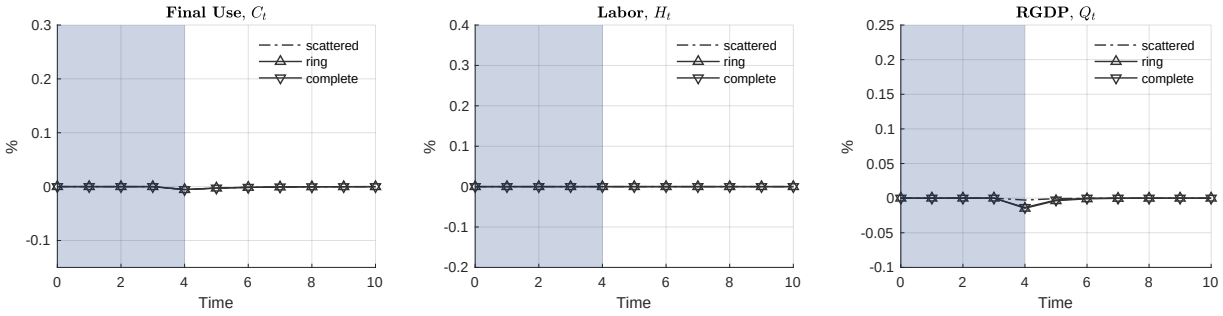


Figure 5: Aggregate responses of three economies with symmetric network structures.

Every series is plotted as percentage deviation from the steady state calculated as log difference. Each horizontal axis represents time t. Shaded areas represent the periods from t_0 to $t_0 + T$. The legends correspond to cross-industry connectedness in each economy. Details are in appendix A.3.

where $\alpha_{\max} = \max_{1 \le i \le n} \alpha_i$. *A representative-firm model can generate the same dynamics of aggregate final uses if* $\widetilde{\alpha} = \boldsymbol{\theta}' \boldsymbol{\alpha}$.

Theorem 3 imposes a strict requirement of the topology of an economy, on both the production side and the consumption side. The condition in equation (4) requires the final-expenditure distribution $\boldsymbol{\theta}$ on the consumption side to be an eigenvector associated with a positive eigenvalue of the input-output matrix on the production side. This essentially requires the two sides to be "isomorphic," in that the same fraction $(1 - \beta\widetilde{\alpha})$ of every firm's gross output is expected to sell to the final consumers. Under this condition, the detailed input-output matrix is not necessary to describe the final-use dynamics of an economy. One number $\widetilde{\alpha}$ is sufficient. In the special case of symmetric final uses with $\boldsymbol{\theta} = \frac{1}{n}\mathbf{1}$, equation (4) is equivalent to requiring identical outdegrees of every order T, such that $\boldsymbol{d}^{\text{out},T} = \widetilde{\alpha}^T \mathbf{1}$. This explains the identical movements of three distinct economies in Figure 5a.

Under condition (4), the law of motion of the aggregate final use C_t satisfies

$$\ln C_t = \widetilde{\alpha} \ln C_{t-1} + \ln z_t + \boldsymbol{\theta}' \boldsymbol{\gamma} \ln A_t + E_t(\widetilde{\alpha}) + \psi^C, \tag{5}$$

where $E_t(\widetilde{\alpha})$ represents the aggregate impact of macro signals, in which $\widetilde{\alpha}$ summarizes the role of the network structure; ψ^C is a constant. Under the same condition, the aggregate real value added $\ln Q_t$ is independent of the full $\boldsymbol{\Omega}$ matrix up to a first-order approximation. If we further impose common input and labor shares in all industries, then the details of $\boldsymbol{\Omega}$ become truly irrelevant for Q_t. This additional restriction forces the outputs of all industries to comove perfectly in response to macro shocks, by equation (3).

Further restrictions are necessary for a representative-firm model to mimic the dynamics of an interconnected economy. If the eigenvalue $\widetilde{\alpha}$ in Theorem 3 equals the final-use weighted average input share $\boldsymbol{\theta}' \boldsymbol{\alpha}$, then a representative-firm model can produce the same dynamics of the aggregate final use described in equation (5). This is the case for the three examples of fully symmetric network structures.

To summarize, the lesson from Theorem 3 and the subsequent analysis is that, generically, the absence of explicit input-output linkages is not without loss when studying the dynamics of the aggregate economy. Exceptions are uncommon.

4.4 Time to Build in the Network

Time to build of inputs is crucial in the model. The common one-period time to build is assumed for simplicity, not a necessity. The mechanism needs only a subset of inputs requiring time to build. Derivation details are in B.1.

Suppose that each firm can purchase some inputs within the same period of production, knowing the output price. These inputs are henceforth called the "just-in-time" inputs. Other inputs (the "time-to-build" inputs) require the same one-period time to build as before. This distinction breaks the input-output matrix into two parts, $\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0$ and $\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1$, where $\boldsymbol{\alpha}_b$ summarizes each industry's share of inputs requiring b-period time to build, and $\boldsymbol{\Omega}_b$ contains the detailed input-output linkages, for $b = 0, 1$, such that $\boldsymbol{\gamma} + \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 \leq \mathbf{1}$ and $\boldsymbol{\Omega}_b\mathbf{1} = \mathbf{1}$. The rest of the model remains unchanged.

In equilibrium, the main changes account for the *intratemporal* input flows in Domar weights $\boldsymbol{s}_t$. The equilibrium dynamics of this economy are characterized by $\boldsymbol{L}_0 \operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1$, where the Leontief inverse of the just-in-time input-output matrix is defined as $\boldsymbol{L}_0 = [\boldsymbol{I} - \operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0]^{-1}$. Intratemporal flows of just-in-time inputs amplify fluctuations through an instantaneous cascade effect, captured by $\boldsymbol{L}_0 \geq \boldsymbol{I}$. The intuition is that any real impact of a signal on industry i transmits immediately to i's suppliers and further-upstream suppliers of just-in-time inputs, thus amplifying the signal's impact. Similarly, the intertemporal shock transmission due to flows of time-to-build inputs downstream spills over to other industries because of just-in-time inputs.

Deriving the counterpart of Theorem 2 with just-in-time inputs requires redefining degrees based on *time*. Given an order T, I redefine the T-th order *weighted indegree* of each industry i as the sum of all incoming walks *to* industry i that take T periods to complete,

$$\widetilde{\boldsymbol{d}^{\text{in},T}} \equiv [\boldsymbol{L}_0 \operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1]^T\mathbf{1}, \quad \text{with} \quad \widetilde{\boldsymbol{d}^{\text{in},T}} \leq \boldsymbol{d}^{\text{in},T}.$$

The inequality indicates that just-in-time inputs weaken the over-time connections between industries. The *weighted outdegrees* are similarly redefined to summarize all possible outgoing walks *from* each industry that take T periods to complete. Unlike the standard degree definition, the redefined and time-based T-th order degrees can involve walks with more than T buyer-seller connections (edges) in the original network.

The average degree $\widetilde{\overline{d^T}}$ and the variance of indegrees $\widetilde{V^{\text{in},T}}$ are counterparts of $\overline{d^T}$ and $V^{\text{in},T}$, respectively. Results similar to those in Theorem 2 can be established. The common component

D_T in the volatility lower bounds has a counterpart $\widetilde{D_T}$, given as

$$(\widetilde{D_T})^2 = (\beta^T)^2 \left(r_A \sigma_A^2 (\widetilde{\overline{d^T}})^2 + r_\theta \sigma_\theta^2 \widetilde{V^{\text{in},T}} \right). \tag{6}$$

As before, $\widetilde{D_T}$ sets the signal-induced volatility lower bounds of all aggregate variables in period t, including the final use $\widehat{c}_t$, labor $\widehat{h}_t$, and the real GDP $\widehat{q}_t$.

The similarity between D_T in Theorem 2 and $\widetilde{D_T}$ above is not a coincidence. Changing the time-to-build assumption alters the equilibrium outcome almost as if by altering the network structure. This is because the impact of horizon-T signals about final uses depends on how many industries and further upstream industries these signals can affect within T periods. In the baseline case, every input-output link takes one period to complete. In contrast, traveling along just-in-time links is always instantaneous. Therefore, this is as if we have a new network structure, where the weights of just-in-time links are appropriately assigned to time-to-build links. After a signal arrives, the same physical and belief-based transmission channels are responsible for the dynamics.

It is ambiguous whether the volatility lower bounds decrease or increase with just-in-time inputs. Although the average degree becomes smaller for sure, $\widetilde{\overline{d^T}} \leq \overline{d^T}$, the dispersion may not. Therefore, introducing just-in-time inputs does not necessarily result in smaller signal-induced fluctuations, and micro signals may play a bigger role than macro ones. Quantitative comparison of models with and without just-in-time inputs is in sections 5.3 and 5.4.

5 Quantitative Illustrations

This section starts with a detailed example of the belief updating process of agents. It involves distributional assumptions and more parameters. Then, the model is parameterized based on the U.S. economy. The numerical exercises aim to illustrate the theoretical results by providing some sense of their quantitative scales.

5.1 Information Structures and Belief Updating Process

The aggregate fundamental A_t follows a log random walk, such that $\ln A_t = \ln A_{t-1} + \eta_t$ with $\eta_t \overset{i.i.d.}{\sim} \mathcal{N}\left(-\frac{1}{2}\sigma_\eta^2, \sigma_\eta^2\right)$. This is also the agents' initial prior belief about the process.[12] A signal

[12] Section 4.1 assumes finite unconditional expectation and variance of $\{A_t\}$ for simplicity. Lower bounds of aggregate volatility analogous to those in Theorem 2 can be established near a given starting point $A_t = A$ as long as the $\{A_t\}$ series has a finite conditional variance such that $\text{var}(A_{t+T} \mid A_t = A) < \infty, \forall T$.

about future $A_{t+\tau}$ is specified as that of $\eta_{t+\tau}$. Let $\nu^t_{t+\tau} \in \mathcal{M}_t$ be a macro signal of $\eta_{t+\tau}$ received in period t, such that $\nu^t_{t+\tau} = \eta_{t+\tau} + \sigma_\epsilon \epsilon^t_{t+\tau}$, with $\epsilon^t_{t+\tau} \overset{i.i.d.}{\sim} \mathcal{N}(0,1)$. The Bayesian updating process is standard. Let $\eta_{t+\tau|t-1}$ and $\sigma^2_{\eta_{t+\tau}|t-1}$ be the agents' period-t prior expectation and variance of $\eta_{t+\tau}$ inherited from period $t-1$, respectively. Given a new signal $\nu^t_{t+\tau}$, the period-t posterior remains normal with expectation and variance given by

$$\eta_{t+\tau|t} = \frac{\sigma^{-2}_{\eta_{t+\tau}|t-1}}{\sigma^{-2}_{\eta_{t+\tau}|t}} \eta_{t+\tau|t-1} + \frac{\sigma^{-2}_\epsilon}{\sigma^{-2}_{\eta_{t+\tau}|t}} \nu^t_{t+\tau}, \quad \text{where} \quad \sigma^{-2}_{\eta_{t+\tau}|t} = \sigma^{-2}_{\eta_{t+\tau}|t-1} + \sigma^{-2}_\epsilon. \tag{7}$$

If $\nu^t_{t+\tau}$ is the first signal about $\eta_{t+\tau}$ that agents ever receive, then the period-t prior is the initial prior with $\eta_{t+\tau|t-1} = -\frac{1}{2}\sigma^2_\eta$ and $\sigma^2_{\eta_{t+\tau}|t-1} = \sigma^2_\eta$. Agents put more weight on relatively more precise signals (higher $\sigma^{-2}_\epsilon / \sigma^{-2}_\eta$). Given the values of $\{\eta_{t+\tau'|t}\}^\tau_{\tau'=1}$, the posterior expectation of $A_{t+\tau}$ satisfies $A_{t+\tau|t} = A_t \exp\left(\sum^\tau_{\tau'=1} \eta_{t+\tau'|t}\right)$.

Let the product-specific preferences vector $\boldsymbol{\theta}_t \in \triangle^{n-1}$ in every period be an i.i.d. draw from a fixed Dirichlet distribution. The Dirichlet distribution has an order n (number of industries) and a scale parameter $\kappa > 0$, such that

$$\boldsymbol{\theta}_t \overset{i.i.d.}{\sim} \text{Dirichlet}_n(\kappa \boldsymbol{\theta}), \quad \text{with } \mathbb{E}[\boldsymbol{\theta}_t] = \boldsymbol{\theta},\ \text{var}(\boldsymbol{\theta}_t) = \frac{1}{\kappa+1}(\text{diag}(\boldsymbol{\theta}) - \boldsymbol{\theta}\boldsymbol{\theta}'). \tag{8}$$

Here κ is an inverse volatility measure of $\boldsymbol{\theta}_t$ and serves as the precision in the initial prior, similar to the role of σ^{-2}_η. I choose the somewhat unconventional Dirichlet distribution for its parsimony in parameters and tractability. The only parameter to be determined is κ, as $\boldsymbol{\theta}$ is directly mapped to average final-expenditure shares in the data. Tractability results from Dirichlet distribution being the conjugate prior of multinomial distributions. Denote the $n \times 1$ vector $\boldsymbol{m}^t_{t+\tau} \in \mathcal{M}_t$ as a micro signal of $\boldsymbol{\theta}_{t+\tau}$ received in period t, such that $\boldsymbol{m}^t_{t+\tau}$ is a draw from a conditional multinomial distribution, satisfying $N\boldsymbol{m}^t_{t+\tau} \mid \boldsymbol{\theta}_{t+\tau} \sim \text{Multinomial}(N, \boldsymbol{\theta}_{t+\tau})$. The precision of each signal is controlled by $N \in \mathbb{N}$. Rescaling $\boldsymbol{m}^t_{t+\tau}$ by N ensures that $\boldsymbol{m}^t_{t+\tau} \in \triangle^{n-1}$ is comparable with $\boldsymbol{\theta}_{t+\tau}$. It follows that $\mathbb{E}[\boldsymbol{m}^t_{t+\tau} \mid \boldsymbol{\theta}_{t+\tau}] = \boldsymbol{\theta}_{t+\tau}$ and $\text{var}(\boldsymbol{m}^t_{t+\tau} \mid \boldsymbol{\theta}_{t+\tau}) = \frac{1}{N}(\text{diag}(\boldsymbol{\theta}_{t+\tau}) - \boldsymbol{\theta}_{t+\tau}\boldsymbol{\theta}'_{t+\tau})$. Let $K_{t+\tau|t-1}$ be the scale parameter for the period-t Dirichlet prior of $\boldsymbol{\theta}_{t+\tau}$, inherited from period $t-1$. The prior expectation is $\boldsymbol{\theta}_{t+\tau|t-1}$. Given a new signal $\boldsymbol{m}^t_{t+\tau}$, period-$t$ posterior remains Dirichlet, with

$$\boldsymbol{\theta}_{t+\tau|t} = \frac{K_{t+\tau|t-1}}{K_{t+\tau|t}} \boldsymbol{\theta}_{t+\tau|t-1} + \frac{N}{K_{t+\tau|t}} \boldsymbol{m}^t_{t+\tau}, \quad \text{where} \quad K_{t+\tau|t} = K_{t+\tau|t-1} + N. \tag{9}$$

If $\boldsymbol{m}^t_{t+\tau}$ is the first signal about $\boldsymbol{\theta}_{t+\tau}$ that agents ever receive, then $K_{t+\tau|t-1} = \kappa$ and $\boldsymbol{\theta}_{t+\tau|t-1} = \boldsymbol{\theta}$.

Similar to macro signals, agents put more weight on more precise signals (higher N/κ) when forming posterior expectations.

I discuss two special cases of the message set specification $\mathcal{M}_t$, henceforth called the *news specification* and the *learning specification*. Each case shares certain features with common assumptions in the existing literature. Let T be the fixed finite horizon of $\mathcal{M}_t$.

News specification. Given T, $\mathcal{M}_t$ under the *news specification* contains exactly one signal about each fundamental in period $t+T$, that is,

$$\mathcal{M}_t^{\text{news}} = \left\{\nu_{t+T}^t, \boldsymbol{m}_{t+T}^t\right\}, \quad \forall t.$$

Such an information structure echos the literature on news-driven business cycles. Agents update their beliefs about future fundamentals *exactly once* before these fundamentals are realized. Their posterior beliefs are thus persistent without further revisions. Posterior precisions are fixed at $\sigma_{t+T|t+\tau}^{-2} = \sigma_\eta^{-2} + \sigma_\epsilon^{-2}$ and $K_{t+T|t+\tau} = \kappa + N$, $\forall \tau = 0, \ldots, T-1$, and posterior expectations satisfy

$$\eta_{t+T|t+\tau} = \frac{-\frac{1}{2} + \sigma_\epsilon^{-2}\nu_{t+T}^t}{\sigma_\eta^{-2} + \sigma_\epsilon^{-2}}, \quad \text{and} \quad \boldsymbol{\theta}_{t+T|t+\tau} = \frac{\kappa\boldsymbol{\theta} + N\boldsymbol{m}_{t+T}^t}{\kappa + N}.$$

Learning specification. Given T, $\mathcal{M}_t$ under the *learning specification* contains one macro signal and one micro signal about every $t+\tau$ with $\tau = 1, \ldots, T$, that is,

$$\mathcal{M}_t^{\text{learning}} = \left\{\nu_{t+\tau}^t, \boldsymbol{m}_{t+\tau}^t\right\}_{\tau=1}^{T}, \quad \forall t.$$

Such an information structure allows agents to accumulate signals over time, as in any models featuring Bayesian learning. Agents receive the first signals about $t+T$ in period t, and keep receiving signals about $t+T$ until period $t+T$ arrives. At any t, agents have accumulated $T+1-\tau$ signals about $t+\tau$ fundamentals and updated relevant beliefs as many times, $\tau = 1, \ldots, T$. Posterior precisions of $\eta_{t+T|t+\tau}$ and $\boldsymbol{\theta}_{t+T|t+\tau}$ increase as $t+T$ approaches, such that $\sigma_{t+T|t+\tau}^{-2} = \sigma_\eta^{-2} + (\tau+1)\sigma_\epsilon^{-2}$ and $K_{t+T|t+\tau} = \kappa + (\tau+1)N$. The posterior expectations keep changing between t and $t+T$ according to

$$\eta_{t+T|t+\tau} = \frac{-\frac{1}{2} + \sigma_\epsilon^{-2}\sum_{\tau'=0}^{\tau}\nu_{t+T}^{t+\tau'}}{\sigma_\eta^{-2} + (\tau+1)\sigma_\epsilon^{-2}}, \quad \text{and} \quad \boldsymbol{\theta}_{t+T|t+\tau} = \frac{\kappa\boldsymbol{\theta} + N\sum_{\tau'=0}^{\tau}\boldsymbol{m}_{t+T}^{t+\tau'}}{\kappa + (\tau+1)N}, \ \tau = 0, \ldots, T-1.$$

When $T = 1$, the *news* and *learning* specifications of $\mathcal{M}_t$ coincide. For $T > 1$, the main dif-

Table 1: Parameters

Fixed parameters			
β	0.99		Quarterly discount factor
A	0.647		Aggregate final-use demand and labor supply
$\boldsymbol{\theta}$	$(\overline{\theta} = 1/71)$		Average final-use expenditure share on each industry†
$\boldsymbol{\gamma}$	$(\overline{\gamma} = 0.305)$		Average ratio of labor compensation to industrial gross output†
$\boldsymbol{\alpha}$	$(\overline{\alpha} = 0.472)$		Average ratio of input expenditure to industrial gross output†
$\boldsymbol{\Omega}$	$(\overline{\omega} = 0.014)$		Average input expenditure distribution by industry†
κ	13738		Volatility in the cross-sectional distribution of final-use expenditures
σ_η	0.0112		Volatility in labor supply and labor compensation share
Model-specific parameters (I: baseline; II: with just-in-time inputs††)			
	I	II	
T	5	5	Forecast horizon in quarters, to match the dynamics of $\boldsymbol{s}_t, \boldsymbol{u}_t$‡
N/κ	0.232	0.232	Precision of product-specific signals, to match the dynamics of $\boldsymbol{s}_t, \boldsymbol{u}_t$‡
$\sigma_\epsilon/\sigma_\eta$	23.66	8.52	Noise in aggregate signals, to match the dynamics of $\boldsymbol{s}_t, \boldsymbol{u}_t$‡

† $\boldsymbol{\theta}$, $\boldsymbol{\gamma}$, and $\boldsymbol{\alpha}$ are 71-by-1 vectors and only their cross-sectional means are shown in the parentheses; $\boldsymbol{\Omega}$ is a 71-by-71 matrix and its all-element mean is shown.

‡ The presented values of (T, N, σ_ϵ) are obtained under the news specification of the message set, $\mathcal{M}_t^{\text{news}}$. Those under the learning specification are in C.4.

†† A set of 16 industries in mining, utilities, construction, and durable manufacturing produces time-to-build inputs. The products of the rest 55 industries are just-in-time inputs.

ference between these two specifications is whether agents can revise their previous beliefs. Both specifications are considered in the quantitative explorations of the model.

5.2 Parameters

The main source of data is the Industry Account of the Bureau of Economic Analysis (BEA). Both the quarterly GDP-by-industry data and the annual input-output data are used. This section briefly discusses the calibration procedure and rationale. Table 1 summarizes the chosen parameters. Details are in C.

The BEA provides information about input-output linkages at three aggregation levels by NAICS codes: sector, summary, and detail levels, with numbers of industries $n = 15$, 71, and 405, respectively. At each aggregation level, the labor input shares $\boldsymbol{\gamma}$ and goods input shares $\boldsymbol{\alpha}$ and $\boldsymbol{\Omega}$ by industry match the average of their data counterparts using the I-O make and use tables.[13] Statistics of network degrees are computed at each aggregation level.

The length of a period is a quarter, which is also the input time to build. This matches the data frequency and it is in line with the estimated range of time to build in manufacturing (3 to 10

[13] I-O annual make and use tables based on NAICS codes are available 1997–2018 at the sector and summary levels and in 2007 and 2012 at the detail level. Government sectors are included in input-output matrices to acknowledge their producer roles. The I-O tables based on the conventional make-use framework in *producer* prices are preferable to those in purchaser prices, and the conventional make-use framework is preferable to the new supply-use framework adopted since 2018. See C.1 for further explanations. Also see C.3 for target choices.

months) by Meier (2018) and in construction (5.5 to 7 months on average for single-family houses) by Oh and Yoon (2020). The discount factor is set to be $\beta = 0.99$. The initial value of A sets the total labor at $1/3$. The number of industries is $n = 71$ to match the summary-level quarterly data. The average final-expenditure share θ_i is the average fraction of final-use expenditure on industry i, $\forall i$.[14] Productivity shocks are not explicitly considered in simulations, thus normalized at $\boldsymbol{z} = \mathbf{1}$. Values of κ and σ_η control the agents' perceived volatility in the fundamentals. Therefore, κ is chosen to match the variance-covariance matrix of final-use expenditure shares and σ_η matches the volatility of changes in hours worked and in labor compensation shares.

When allowing for just-in-time inputs, I assume that the products of mining, utilities, construction, and durable manufacturing remain time-to-build inputs. The two sets of inputs are mutually exclusive, for simplicity. Fine-tuning this assumption needs a detailed estimation of time to build for every buyer-seller pair of inputs, which is beyond the scope of this paper. The simple assumption here aims to capture the idea that the set of time-to-build inputs is nonempty but can be relatively small.

Forecast horizon T and precisions of signals N and σ_ϵ govern posterior expectations, which do not have direct data counterparts. In the model, these parameters control the updating speed and persistence of agents' beliefs, and so they affect the volatility in firms' decisions and the correlations between these decisions over time and across industries. The mechanism relies on the chains of input needs, and the most relevant variables are the Domar weights $\boldsymbol{s}_t$ and each industry's input expenditure $\boldsymbol{u}_t \equiv (u_{i,t}) = \left(\frac{\sum_j p_{j,t} x_{ij,t}}{P_t C_t}\right)$. Therefore, T, N, and σ_ϵ are picked using the method of simulated moments to match the volatility and one-period cross-industry serial correlation in $\boldsymbol{s}_t$ and $\boldsymbol{u}_t$. Further details are in C.4. The unit-free data counterparts of $\boldsymbol{s}_t$ and $\boldsymbol{u}_t$ are available at quarterly frequency (2005Q1–2019Q4). Per the model, $\boldsymbol{s}_t$ and $\boldsymbol{u}_t$ are less responsive to other shocks, including productivity shocks. Admittedly, the parsimonious nature of the Dirichlet-multinomial assumption of micro shocks and signals limits the model's capability to match a large number of industry-level moments. Outside of the model, other forces also drive changes in $\boldsymbol{s}_t$ and $\boldsymbol{u}_t$, especially any change in the network structure itself in the long run. I argue that the impact of network change is minimal given the relatively short sample length. And to control for any time trend, the moments are calculated based on the first-order differences of each series.[15]

[14] The measure of total final use is GDP less net export. B.2 describes the import and export treatment in the model and C.2 discusses additional parameters. The final use includes government expenditures. B.3 shows an alternative treatment of the government in the model. The final use also includes investments. C.5 contains further discussion about capital flows.

[15] Other forces are quantitatively important especially for the high volatility in $\boldsymbol{u}_t$. Low elasticity of input substitu-

Four sets of (T, N, σ_ϵ) are separately picked for both versions of the model (baseline and with just-in-time inputs) and both specifications of $\mathcal{M}_t$. The *news* specification appears to fit the data moments better than the *learning* specification in both versions. Table 1 reports the two sets of parameter values under the *news* specification. The others are in the appendix. The four sets of parameter values are consistent, in the following sense. First, the signal horizon T remains well above 1, ranging from 5 to 8 quarters. In the model, this means that agents receive signals about future fundamentals several periods before their realization. Higher-order indegrees $\boldsymbol{d}^{\text{in},\tau}$, $\tau > 1$, are therefore informative about aggregate fluctuations caused by these signals. Second, these signals, macro or micro, remain quite noisy in comparison with the prior distributions. This is captured by $N/\kappa < 1$ for micro signals and $\sigma_\epsilon^{-2}/\sigma_\eta^{-2} < 1$ for macro signals. In the model, this means slow updating and high persistence of beliefs. Third, relatedly, the *learning* specification favors even more noise in each signal. This is intuitive as *learning* allows signal accumulation when $T > 1$ which improves belief accuracy over time. Signals must be noisier in *learning* than in *news* for them to generate similar dynamics.

5.3 Statistics of Network Degrees at Various Aggregation Levels

This section addresses two concerns. The first is whether the cross-sectional means $\overline{d^\tau}$ and variances $V^{\text{in},\tau}$ of indegrees $\{\boldsymbol{d}^{\text{in},\tau}\}_{\tau=1}^{T}$ change as the number of industries n increases. If these statistics change in n, then the magnitude of signal-induced fluctuations likely differs at each aggregation level. A particular aspect of this concern is whether $V^{\text{in},\tau}$ and consequently the impact of micro signals diminishes as $n \to \infty$. The second concern regards the time-to-build assumption. Allowing for just-in-time inputs has a similar effect on the economy to altering the network structure. It is therefore worth examining how much the mean degrees decrease and whether the variances change with just-in-time inputs.

Table 2 reports the cross-sectional mean and standard deviation of indegrees of the first few orders, at three data aggregation levels. Panel I uses standard weighted indegrees defined by the full matrix $\text{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}$ as in section 4.1. Panel II illustrates what changes with just-in-time inputs, using redefined and time-based indegrees discussed in section 4.4. As an example, Figure 6

tion at least at the relatively high sectoral aggregation level leads to high input volatility, as estimated by Atalay (2017). Time-varying factor utilization also affects input uses, first studied by Basu (1996). Material inputs are complements to capital and labor. Thus when the utilization rate of capital and labor varies, expenditures on material inputs fluctuate. Adjustment costs, inventory costs, or temporary financial constraints also contribute to lumpy and volatile input expenditures. Including the less volatile $\boldsymbol{s}_t$ in the objective function helps to avoid attributing too much of the high volatility in input expenditures to changes in expectations.

Table 2: Indegree statistics of order τ for input-output networks at three aggregation levels

Aggregation level	n	$\tau=1$		$\tau=2$		$\tau=3$		$\tau=4$	
		$\overline{d^\tau}$	$\sqrt{V^{\text{in},\tau}}$	$\overline{d^\tau}$	$\sqrt{V^{\text{in},\tau}}$	$\overline{d^\tau}$	$\sqrt{V^{\text{in},\tau}}$	$\overline{d^\tau}$	$\sqrt{V^{\text{in},\tau}}$
I. Baseline model ($\boldsymbol{d}^{\text{in},\tau}$)									
Sector	15	0.432	0.092	0.200	0.062	0.095	0.035	0.056	0.018
Summary	71	0.472	0.140	0.234	0.088	0.115	0.049	0.057	0.026
Detail‡	402	0.535	0.162	0.296	0.126	0.160	0.081	0.085	0.047
II. Allowing for just-in-time inputs† ($\widetilde{\boldsymbol{d}^{\text{in},\tau}}$)									
Sector	15	0.181	0.107	0.074	0.045	0.031	0.019	0.013	0.009
Summary	71	0.158	0.131	0.052	0.050	0.019	0.021	0.007	0.008
Detail‡	402	0.237	0.158	0.087	0.076	0.035	0.035	0.014	0.015

Panel I contains statistics of indegrees $\boldsymbol{d}^{\text{in},\tau}$ as discussed in section 4.1, and Panel II shows those of $\widetilde{\boldsymbol{d}^{\text{in},\tau}}$ as in section 4.4.

† At the sector level, 4 industries produce time-to-build inputs: mining (BEA IO code 21), utilities (code 22), construction (code 23), and manufacturing (code 31G). At the summary and detail levels, products of nondurable good manufacturing are considered just-in-time inputs and the rest remains unchanged. Under this assumption, 16 summary-level industries and 187 detail-level industries produce time-to-build inputs. For each industry j, $\alpha_{i,0}\omega_{ij,0} + \alpha_{i,1}\omega_{ij,1} = \alpha_i\omega_{ij}$ in every industry i's production function. If product j serves as a time-to-build input, then $\alpha_{i,0}\omega_{ij,0} = 0$, $\forall i$; similarly, if j's product is a just-in-time input, then $\alpha_{i,1}\omega_{ij,1} = 0$, $\forall i$.

‡ Only industries ($n = 402$) that have positive total outputs in the I-O tables are included for this exercise.
In the Supplementary Materials, Figure D.1 plots the same statistics based on the input-output matrix in each sample year.

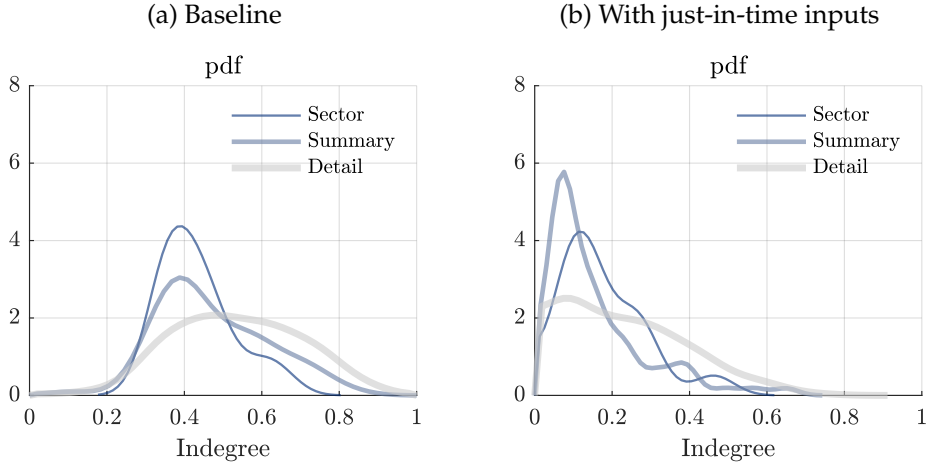


Figure 6: Cross-sectional distribution of indegrees of order $\tau = 1$ at each aggregation level.
At each aggregation level, Figure 6a plots the kernel density estimate of the cross-sectional distribution of first order weighted indegrees $\{d_i^{\text{in},1}\}_{i=1}^n$ defined using the full input-output matrix, and Figure 6b plots the same for redefined and time-based first order weighted indegrees $\{\widetilde{d_i^{\text{in},1}}\}_{i=1}^n$ after adjusting for just-in-time inputs.
D contains more figures for degree distributions.

compares the density functions of the cross-sectional distribution of the first order indegrees (i.e., good input shares $\{\alpha_i\}_{i=1}^n$) at each aggregation level.

Comparing values of $\overline{d^\tau}$ and $V^{\text{in},\tau}$ across aggregation levels addresses the first concern. Micro signals have diminishing impacts only if $V^{\text{in},\tau}$ decreases in n for at least some τ. However, with or without just-in-time inputs, neither $\overline{d^\tau}$ nor $V^{\text{in},\tau}$ appears to decline as n increases, $\forall\tau$. The most disaggregated level actually has the largest values of $\overline{d^\tau}$ and $V^{\text{in},\tau}$, as the distribution of $d_i^{\text{in},\tau}$ is more dispersed with a higher mean (Figure 6a). Hence, a larger network size does not necessarily lead to smaller aggregate impacts of micro signals.

As to the second concern regarding time to build, it is as expected that $\overline{\widetilde{d^{\text{in},\tau}}}$ is lower for every τ. The decline is significant due to the small set of remaining time-to-build inputs. However, the cross-sectional variation $V^{\text{in},\tau}$ among these degrees is not necessarily smaller, or smaller by a similar proportion. This is because the distribution of $\widetilde{d_i^{\text{in},\tau}}$ has a relatively bigger mass near the right end, at each aggregation level (Figure 6b). Therefore, in this particular exercise, the impact scale of a given micro signal is less sensitive to the time-to-build specification than that of a macro signal. Nonetheless, these results suggest that the time dimension of production technologies is critical in the study of input-output linkages.

5.4 Aggregate Business-Cycle Moments

This section illustrates the effect of the information structure and the time-to-build restriction on the aggregate dynamics using business-cycle moments. The intention is not to reproduce the moment values as in the data, which requires further enriching the stylized model. Rather, this exercise aims to provide a quantitative scale for theoretical analysis.

Table 3 compares a few business-cycle moments generated by various versions of the model. Data moments are included for reference. Parameters are fixed in each panel. Overall, when both macro and micro shocks and signals are present, each version of the model produces second-moment business-cycle statistics comparable to their data counterparts. Without productivity shocks, macro variables tend to fluctuate less, especially with just-in-time inputs. Labor is volatile and almost perfectly correlates to GDP because of the infinite Frisch elasticity of labor supply and the demand-side origin of all shocks. With unitary elasticity of substitution among inputs and among final uses in a frictionless environment, aggregate input use shows similar volatility to output and final use .[16]

[16]Plausible extensions of the model can improve its quantitative performance at the cost of tractability. Examples

Table 3: Simulated business-cycle moments under various model specifications, based on % deviation from trend

	Real GDP (Q)		Real Final Use (C)			Hours (H)			Real Input (X)		
Specification of $\mathcal{M}_t$ †	σ_Q	ρ_Q	σ_C/σ_Q	$\rho_{C,Q}$	ρ_C	σ_H/σ_Q	$\rho_{H,Q}$	ρ_H	σ_X/σ_Q	$\rho_{X,Q}$	ρ_X
Data	1.12	0.88	1.33	0.98	0.89	1.29	0.89	0.83	3.82	0.78	0.84
I. Baseline model ($T = 5,\ N/\kappa = 0.232,\ \sigma_\epsilon/\sigma_\eta = 23.66$)‡											
News	0.80	0.69	0.87	0.92	0.77	1.84	1.00	0.70	0.88	0.92	0.77
Learning	0.80	0.69	0.88	0.92	0.77	1.84	1.00	0.70	0.89	0.91	0.78
Perfect foresight	0.70	0.80	1.36	0.91	0.57	1.82	1.00	0.81	1.33	0.54	0.72
No signal	0.80	0.69	0.87	0.92	0.77	1.84	1.00	0.69	0.87	0.92	0.77
II. With just-in-time inputs ($T = 5,\ N/\kappa = 0.232,\ \sigma_\epsilon/\sigma_\eta = 8.52$)‡											
News	0.55	0.64	0.97	0.95	0.66	2.76	0.99	0.66	1.15	0.80	0.55
Learning	0.55	0.64	0.98	0.95	0.65	2.77	0.99	0.66	1.43	0.61	0.43
Perfect foresight	0.54	0.65	1.07	0.95	0.58	2.74	0.99	0.68	2.21	0.07	0.34
No signal	0.55	0.64	0.97	0.95	0.66	2.76	0.99	0.66	0.97	0.95	0.66

All data and model variables are presented in the percentage deviation from trend calculated by applying the Hodrick-Prescott (HP) filter with a smoothing parameter of 1600 to their logarithms. Model-generated statistics are based on simulations of 5000 periods starting from the corresponding steady state. The first 500 periods are discarded. All simulations share a common history of fundamental realizations $\{A_t, \boldsymbol{\theta}_t\}_{t=1}^{5000}$. For every variable X, σ_X is its standard deviation, $\rho_{X,Q}$ its correlation coefficient to real GDP (Y), and ρ_X its first order autocorrelation.

† Section 5.1 defines the news and learning specifications of the message set $\mathcal{M}_t$. Perfect foresight means that agents can perfectly foresee the actual future fundamental realizations. No signal means that agents do not receive additional information and their decisions are based on the prior beliefs.

‡ Parameter values are obtained under the *news* specification. They do not change across simulations within each panel I or II. Hence, in each panel, $\mathcal{M}_t^{\text{learning}}$ simply contains more signals of the same precision than $\mathcal{M}_t^{\text{news}}$. Results based on alternative parameter values obtained under the *learning* specification are in Table D.1 of the Supplementary Materials.

Table 4: Business-cycle moments generated by *micro* shocks only, based on % deviation from trend

	Real GDP (Q)		Real Final Use (C)			Hours (H)			Real Input (X)		
Specification of $\mathcal{M}_t$	σ_Q	ρ_Q	σ_C/σ_Q	$\rho_{C,Q}$	ρ_C	σ_H/σ_Q	$\rho_{H,Q}$	ρ_H	σ_X/σ_Q	$\rho_{X,Q}$	ρ_X
I. Baseline model, micro only ($T = 5,\ N/\kappa = 0.232$)											
News	0.18	−0.02	1.14	0.39	−0.17	1.61	0.92	−0.05	1.20	0.39	−0.03
Learning	0.17	−0.01	1.17	0.43	−0.17	1.69	0.94	−0.03	1.33	0.45	0.14
Perfect foresight	0.18	−0.04	0.97	0.55	−0.14	2.41	0.92	−0.06	0.97	0.55	−0.14
No signal	0.18	−0.03	1.13	0.36	−0.17	1.58	0.92	−0.06	1.13	0.36	−0.17
II. With just-in-time inputs, micro only ($T = 5,\ N/\kappa = 0.232$)											
News	0.18	−0.04	0.99	0.53	−0.15	2.42	0.93	−0.06	2.04	0.27	0.20
Learning	0.17	−0.05	1.00	0.51	−0.17	2.47	0.93	−0.07	3.26	0.19	0.22
Perfect foresight	0.17	−0.05	1.04	0.47	−0.20	2.52	0.94	−0.08	4.44	0.16	0.25
No signal	0.18	−0.04	0.97	0.55	−0.14	2.41	0.92	−0.06	0.97	0.55	−0.14

See notes of Table 3. "Micro only" means that all agents know for sure $A_t = A, \forall t$. The only source of uncertainty and fluctuations is the micro fundamental shocks, and each $\mathcal{M}_t$ contains micro signals only, depending on the message set specification. The history of micro fundamental realizations $\{\boldsymbol{\theta}_t\}_{t=1}^{5000}$ is the same as before, and so are the micro signal realizations in the simulation of every model variant.

More accurate information does not mean smaller fluctuations when fundamentals are volatile. The noisiness of signals slows down belief changes of rational agents. Therefore, model-generated moments under *news* and *learning* specifications of $\mathcal{M}_t$ are very similar to each other, and both are closer to the extreme of no signal than to the other extreme of foreseeing future fundamentals with no uncertainty. The difference between the two extremes indicates the potential room for the information structure to affect measurable variables. As the variable directly affected, the real total input series at the two extremes have a correlation coefficient of 0.22, even with a common history of fundamental realizations.

These moments also illustrate the role of time to build in affecting aggregate volatility. Fluctuations in real GDP and final use are smaller with just-in-time inputs, mainly because of the smaller *average* degrees in Table 2. The real total input appears to be less affected by this change. Indegree *dispersions* in Table 2 predict that allowing for just-in-time inputs should not much affect the impact of micro shocks. Table 4 shows the same moments when the aggregate shocks are muted. In both versions of the model, micro shocks are responsible for a substantial portion of aggregate fluctuations, more so with just-in-time inputs. The role of information structure is also more prominent, as the difference between the two extremes shows. This confirms the previous discussion that a small set of time-to-build inputs is sufficient to create intertemporal connections, and just-in-time inputs serve to amplify the impacts of both realized and yet-to-be-realized disturbances.

6 Conclusion

Input-output linkages facilitate expectations-driven fluctuations because inputs take time to build. At the rational-expectations competitive equilibrium, chains of input needs over time not only transmit realized disturbances downstream but also open channels for noisy signals about future fundamentals to affect current outcomes. Given the characteristics of signals, simple network statistics capture the role of the input-output network structure and the time-to-build specification in determining the magnitude of aggregate fluctuations in response to both macro and micro signals. In general, the absence of an explicit input-output network in a model is not without loss.

The combined importance of the interindustry network structure and time to build suggests directions for future research. One is a systematic estimation of time to build by commodity, industry, and region. It would also shed light on related topics such as supply-chain disruptions

include input factor adjustment costs, inventory and capital accumulation, varying primary input utilization rate, and low elasticity of input substitution, etc. The mechanism does not change qualitatively.

and regional heterogeneity. Another is to use a quantitatively enriched version of the model to evaluate the industry-specific effects of policy foresight and subsequent transmission. A given fiscal or monetary policy's efficacy will likely depend on the network structure, time to build, and the foresight horizon (e.g., the time between policy announcement and its implementation). This model can be modified to accommodate all these factors.

References

Acemoglu, D., Akcigit, U., Kerr, W., 2016. Networks and the macroeconomy: An empirical exploration. NBER Macroeconomics Annual 30, 273–335.

Acemoglu, D., Carvalho, V.M., Ozdaglar, A., Tahbaz-Salehi, A., 2012. The network origins of aggregate fluctuations. Econometrica 80, 1977–2016.

Atalay, E., 2017. How important are sectoral shocks? American Economic Journal: Macroeconomics 9, 254–80.

Atalay, E., Hortacsu, A., Roberts, J., Syverson, C., 2011. Network structure of production. Proceedings of the National Academy of Sciences 108, 5199–5202.

Baqaee, D.R., 2018. Cascading failures in production networks. Econometrica forthcoming.

Baqaee, D.R., Farhi, E., 2019. The macroeconomic impact of microeconomic shocks: beyond hulten's theorem. Econometrica 87, 1155–1203.

Barsky, R.B., Sims, E.R., 2011. News shocks and business cycles. Journal of Monetary Economics 58, 273–289.

Barsky, R.B., Sims, E.R., 2012. Information, animal spirits, and the meaning of innovations in consumer confidence. American Economic Review 102, 1343–77.

Bartelme, D., Gorodnichenko, Y., 2015. Linkages and economic development. Technical Report. National Bureau of Economic Research.

Basu, S., 1996. Procyclical productivity: increasing returns or cyclical utilization? The Quarterly Journal of Economics 111, 719–751.

Beaudry, P., Portier, F., 2004. An exploration into pigou's theory of cycles. Journal of Monetary Economics 51, 1183–1216.

Beaudry, P., Portier, F., 2006. Stock prices, news, and economic fluctuations. American Economic Review 96, 1293–1307.

Beaudry, P., Portier, F., 2007. When can changes in expectations cause business cycle fluctuations in neo-classical settings? Journal of Economic Theory 135, 458–477.

Beaudry, P., Portier, F., 2014. News-driven business cycles: Insights and challenges. Journal of Economic Literature 52, 993–1074.

Bloch, F., Jackson, M.O., Tebaldi, P., 2019. Centrality measures in networks. Available at SSRN 2749124 .

Carvalho, V., Gabaix, X., 2013. The great diversification and its undoing. American Economic Review 103, 1697–1727.

Carvalho, V.M., 2010. Aggregate fluctuations and the network structure of intersectoral trade. working paper. University of Cambridge.

Christiano, L.J., Motto, R., Rostagno, M., 2014. Risk shocks. American Economic Review 104, 27–65.

Den Haan, W.J., Kaltenbrunner, G., 2009. Anticipated growth and business cycles in matching models. Journal of Monetary Economics 56, 309–327.

Dupor, B., 1999. Aggregation and irrelevance in multi-sector models. Journal of Monetary Economics 43, 391–409.

Foerster, A.T., Sarte, P.D.G., Watson, M.W., 2011. Sectoral versus aggregate shocks: A structural factor analysis of industrial production. Journal of Political Economy 119, 1–38.

Gabaix, X., 2011. The granular origins of aggregate fluctuations. Econometrica 79, 733–772.

Hansen, G.D., 1985. Indivisible labor and the business cycle. Journal of Monetary Economics 16, 309–327.

Horvath, M., 1998. Cyclicality and sectoral linkages: Aggregate fluctuations from independent sectoral shocks. Review of Economic Dynamics 1, 781–808.

Horvath, M., 2000. Sectoral shocks and aggregate fluctuations. Journal of Monetary Economics 45, 69–106.

Jaimovich, N., Rebelo, S., 2009. Can news about the future drive the business cycle? American Economic Review 99, 1097–1118.

Karen J. Horowitz, M.A.P., 2009. Concepts and Methods of the Input-Output Accounts. U.S. Bureau of Economic Analysis (BEA).

Kydland, F.E., Prescott, E.C., 1982. Time to build and aggregate fluctuations. Econometrica: Journal of the Econometric Society , 1345–1370.

Long, J.B.J., Plosser, C.I., 1983. Real business cycles. Journal of Political Economy 91, 39–69.

Meier, M., 2018. Time to Build and the Business Cycle. mimeo. University of Mannheim.

Oh, H., Yoon, C., 2020. Time to build and the real-options channel of residential investment. Journal of Financial Economics 135, 255 – 269.

Pigou, A.C., 1927. Industrial fluctuations. Second ed., Macmillan and Company, limited.

Shea, J., 2002. Complementarities and comovements. Journal of Money, Credit and Banking , 412–433.

Supplementary Materials

A Baseline Model: Proofs and Other Details

A.1 Details of Section 3 and Proof of Theorem 1

A.1.1 Rational-Expectations Competitive Equilibrium

Given market prices and industry i's decisions, the continuation value of industry i's expected profit stream at time t is given as

$$\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\lambda_{t,t+\tau}\pi_{i,t+\tau}\right]=\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\lambda_{t,t+\tau}\left(p_{i,t+\tau}y_{i,t+\tau}-w_{t+\tau}h_{i,t+\tau}-\sum_{j=1}^{n}p_{j,t+\tau}x_{ij,t+\tau}\right)\right],$$

where $\mathbb{E}_t[\cdot]$ is the expectation operator conditional on available information at t, $\lambda_{t,t+\tau}$ is the stochastic discount factor between t and $t+\tau$. The equilibrium stochastic discount factor $\lambda_{t,t+\tau}$ at which firms discount the future profit stream is given by $\lambda_{t,t+\tau}=\beta^{\tau}(A_{t+\tau}\sum_i p_{i,t}c_{i,t})/(A_t\sum_i p_{i,t+\tau}c_{i,t+\tau})$, $\forall\tau\geq 1$.

Industry i's optimization condition for input purchases at t evaluated at the competitive equilibrium can be written as

$$p_{j,t}x_{ij,t}=\alpha_i\omega_{ij}\mathbb{E}_t[\lambda_{t,t+1}p_{i,t+1}y_{i,t+1}],\quad\forall j,$$

where the equilibrium one-period-ahead stochastic discount factor is $\lambda_{t,t+1}=\beta\frac{A_{t+1}}{A_t}\frac{\sum_i p_{i,t}c_{i,t}}{\sum_i p_{i,t+1}c_{i,t+1}}=\beta\frac{A_{t+1}\boldsymbol{p}_t'\boldsymbol{c}_t}{A_t\boldsymbol{p}_{t+1}'\boldsymbol{c}_{t+1}}$. Plugging in the definitions of $s_{i,t}=\frac{p_{i,t}y_{i,t}}{\boldsymbol{p}_t'\boldsymbol{c}_t}$ and $u_{ij,t}=\frac{p_{j,t}x_{ij,t}}{\boldsymbol{p}_t'\boldsymbol{c}_t}$ gives

$$u_{ij,t}=\beta\alpha_i\omega_{ij}\mathbb{E}_t[A_{t+1}s_{i,t+1}]/A_t.$$

The market-clearing condition for good j is

$$y_{j,t}=c_{j,t}+\sum_{i=1}^{n}x_{ij,t};$$

multiplied on both sides by price $p_{j,t}$ and adjusted for total final-use expenditure, it becomes

$$s_{j,t}=\theta_{j,t}+\beta\sum_{i=1}^{n}\alpha_i\omega_{ij}\mathbb{E}_t[A_{t+1}s_{i,t+1}]/A_t.$$

Equation (1) contains the same equations in vector form.

The production function for industry i can be written in logarithms such that

$$\ln y_{i,t}=\ln z_{i,t}+\gamma_i\ln h_{i,t}+\alpha_i\sum_{j=1}^{n}\omega_{ij}\ln x_{ij,t-1}.$$

The household's labor supply condition $H_t = A_t \frac{w_t H_t}{\boldsymbol{p}_t' \boldsymbol{c}_t}$ combined with each industry's labor demand condition $w_t h_{i,t} = \gamma_i p_{i,t} y_{i,t}$, $\forall i$, yields $\ln h_{i,t} = \ln A_t + \ln \gamma_i + \ln s_{i,t}$. Meanwhile, as good markets-clearing conditions imply,

$$x_{ij,t-1} = y_{j,t-1} \frac{\beta \alpha_i \omega_{ij} \mathbb{E}_{t-1}[A_t s_{i,t}]}{A_{t-1} s_{j,t-1}}.$$

Therefore, the equilibrium production level evolves according to

$$\ln y_{i,t} = \ln z_{i,t} + \gamma_i \ln A_t + \gamma_i \ln s_{i,t} + \alpha_i \left(\sum_j \omega_{ij} (\ln y_{j,t-1} - \ln s_{j,t-1}) + \ln(\mathbb{E}_{t-1}[A_t s_{i,t}] / A_{t-1}) \right) + \psi_i,$$

where ψ_i is a constant such that $\psi_i = \gamma_i \ln \gamma_i + \alpha_i \ln \beta + \alpha_i \ln \alpha_i + \alpha_i \sum_j \omega_{ij} \ln \omega_{ij}$. Equation (3) is the vector form of the system.

I use the fixed basket $\boldsymbol{\theta}$ to define the final-use compound C_t and the associated price index P_t, which satisfy

$$\ln C_t = \boldsymbol{\theta}' \ln \boldsymbol{c}_t \quad \text{and} \quad \ln P_t = \boldsymbol{\theta}' (\ln \boldsymbol{p}_t - \ln \boldsymbol{\theta}_t).$$

It is straightforward to verify that $P_t C_t = \boldsymbol{p}_t' \boldsymbol{c}_t$ because the household optimization condition implies that $\frac{p_{i,t} c_{i,t}}{\theta_{i,t}} = \frac{p_{j,t} c_{j,t}}{\theta_{j,t}}$, $\forall\ i, j$. It also indicates that I can use the preference-based time-varying basket $\boldsymbol{\theta}_t$ to define C_t and P_t. Theoretical results are unaffected. Numerical results do differ because $\boldsymbol{\theta}_t$ automatically results in time-varying C_t.

To match the (real) GDP accounting rule in practice, the real aggregate value added between $t-1$ and t should be

$$\begin{aligned} Q_t =& \sum_i \frac{p_{i,t}}{P_t} y_{i,t} - \sum_i \sum_j \frac{p_{j,t}}{P_t} x_{ij,t-1} \\ =& C_t \sum_i s_{i,t} - \sum_i \frac{p_{i,t}}{P_t} (y_{i,t-1} - c_{i,t-1}) \\ =& C_t \sum_i s_{i,t} - C_{t-1} \sum_i \frac{p_{i,t}/P_t}{p_{j,t-1}/P_{t-1}} (s_{i,t-1} - \theta_{i,t-1}). \end{aligned}$$

Note that $\frac{p_{i,t}/P_t}{p_{j,t-1}/P_{t-1}} = \frac{C_t s_{i,t}/y_{i,t}}{C_{t-1} s_{i,t-1}/y_{i,t-1}}$, and so

$$Q_t = C_t \left(\sum_i s_{i,t} - \sum_i \frac{s_{i,t}/s_{i,t-1}}{y_{i,t}/y_{i,t-1}} (s_{i,t-1} - \theta_{i,t-1}) \right).$$

A.1.2 Proof of Theorem 1

Iterating the equilibrium Euler equation (1) gives

$$\boldsymbol{s}_t = \boldsymbol{\theta}_t + \frac{1}{A_t} \beta \boldsymbol{\Omega}' \operatorname{diag}(\boldsymbol{\alpha}) \mathbb{E}_t \left[A_{t+1} \boldsymbol{s}_{t+1} \right]$$

$$
\begin{aligned}
=&\boldsymbol{\theta}_t + \frac{1}{A_t}\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha})\mathbb{E}_t\left[A_{t+1}\left(\boldsymbol{\theta}_{t+1} + \beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha})\frac{1}{A_{t+1}}\mathbb{E}_{t+1}[A_{t+2}\boldsymbol{s}_{t+1}]\right)\right]\\
=&\boldsymbol{\theta}_t + \frac{1}{A_t}\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha})\mathbb{E}_t\left[A_{t+1}\boldsymbol{\theta}_{t+1}\right]\\
&+ \frac{1}{A_t}(\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^2\mathbb{E}_t\left[A_{t+1}\frac{1}{A_{t+1}}\mathbb{E}_{t+1}\left[A_{t+2}\boldsymbol{\theta}_{t+2}\right]\right]\\
&+ \frac{1}{A_t}(\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^3\mathbb{E}_t\left[A_{t+1}\frac{1}{A_{t+1}}\mathbb{E}_{t+1}\left[A_{t+2}\frac{1}{A_{t+2}}\mathbb{E}_{t+2}\left[A_{t+3}\boldsymbol{\theta}_{t+3}\right]\right]\right]\\
&+ \ldots.
\end{aligned}
$$

Equation (2) follows after applying the law of iterated expectations. Separability comes from the independence assumption on the random variables.

For $\boldsymbol{s}_t$ to be well-defined, a sufficient condition is that $\mathbf{1}'\boldsymbol{s}_t < \infty$, because every component to build $\boldsymbol{s}_t$ is element-wise non-negative. Under Assumption 1, $A_{t+\tau|t}/A_t \leq \overline{A}\beta^{-\tau}$. Moreover, let $\alpha_{\max} \equiv \max_i \alpha_i$, the input-output matrix has the following property: $\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}\mathbf{1} = \boldsymbol{\alpha} \leq \alpha_{\max}\mathbf{1}$, and so $(\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega})^\tau\mathbf{1} = (\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega})^{\tau-1}(\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}\mathbf{1}) \leq \alpha_{\max}(\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega})^{\tau-1}\mathbf{1} \leq \ldots \leq (\alpha_{\max})^\tau\mathbf{1}$. It follows that

$$
\begin{aligned}
\mathbf{1}'\boldsymbol{s}_t \leq& 1 + \overline{A}\sum_{\tau=1}^{\infty}\mathbf{1}'(\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^\tau\boldsymbol{\theta}_{t+\tau|t} = 1 + \overline{A}\sum_{\tau=1}^{\infty}[(\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega})^\tau\mathbf{1}]'\boldsymbol{\theta}_{t+\tau|t}\\
\leq& 1 + \overline{A}\sum_{\tau=1}^{\infty}(\alpha_{\max})^\tau = 1 + \frac{\alpha_{\max}\overline{A}}{1-\alpha_{\max}} < \infty.
\end{aligned}
$$

A.2 Details of Section 4.1 and Proof of Theorem 2

In equilibrium, total output is split between final uses and input uses, with $\frac{c_{i,t}}{y_{i,t}} = \frac{\theta_{i,t}}{s_{i,t}}$, and so $\ln \boldsymbol{c}_t = \ln \boldsymbol{y}_t + \ln \boldsymbol{\theta}_t - \ln \boldsymbol{s}_t$. The logarithms of the aggregate variables can then be written as

$$
\begin{aligned}
\ln C_t =& \boldsymbol{\theta}'(\ln \boldsymbol{y}_t + \ln \boldsymbol{\theta}_t - \ln \boldsymbol{s}_t),\\
\ln Q_t =& \ln C_t + \ln\left(\mathbf{1}'\boldsymbol{s}_t - (\boldsymbol{s}_{t-1} - \boldsymbol{\theta}_{t-1})'[(\boldsymbol{s}_t \oslash \boldsymbol{s}_{t-1}) \oslash (\boldsymbol{y}_t \oslash \boldsymbol{y}_{t-1})]\right),\\
\ln H_t =& \ln A_t + \ln(\boldsymbol{\gamma}'\boldsymbol{s}_t),
\end{aligned}
$$

where I use $\oslash$ for element-wise division such that $\boldsymbol{x} \oslash \boldsymbol{y} = \operatorname{diag}(\boldsymbol{y})^{-1}\boldsymbol{x}$, $\forall\, \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$ with $y_i \neq 0$, $\forall 1 \leq i \leq n$.

A.2.1 Stationary Domar weights and outdegrees

A deterministic steady state (SS) exists when A_t is fixed at A and $\boldsymbol{\theta}_t$ at $\frac{1}{n}\mathbf{1}$. The steady-state Domar-weight vector satisfies $\boldsymbol{s} = \boldsymbol{\theta} + \beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{s}$, or

$$
\boldsymbol{s} = \frac{1}{n}[\boldsymbol{I} - \beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha})]^{-1}\mathbf{1}.
$$

Note that the biggest eigenvalue of the (transpose of) input-output matrix $\mathbf{\Omega}'\operatorname{diag}(\boldsymbol{\alpha})$ cannot exceed $\alpha_{\max} \equiv \max_i \alpha_i \leq 1-\gamma$. So we can rewrite $\boldsymbol{L}' = [\boldsymbol{I} - \beta\mathbf{\Omega}'\operatorname{diag}(\boldsymbol{\alpha})]^{-1}$ as

$$\boldsymbol{L}' = \boldsymbol{I} + \beta\mathbf{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}) + (\beta\mathbf{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^2 + (\beta\mathbf{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^3 + \dots.$$

Then, by definition of outdegree of each order,

$$\begin{aligned} n\boldsymbol{s} =& [\boldsymbol{I} + \beta\mathbf{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}) + (\beta\mathbf{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^2 + (\beta\mathbf{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^3 + \dots]\mathbf{1} \\ =& \mathbf{1} + \beta\boldsymbol{d}^{\text{out},1} + \beta^2\boldsymbol{d}^{\text{out},2} + \beta^3\boldsymbol{d}^{\text{out},3} + \dots = \boldsymbol{d}^{\text{out, all}}. \end{aligned}$$

A.2.2 Proof of Theorem 2: lower bounds of aggregate volatility

Here I consider a slightly more general case with $\operatorname{var}(\theta_{i,t}) = n^{-\delta}\sigma_\theta^2$, for some δ. This allows for a clearer picture of the direct role of n in determining the volatility lower bounds. The common component in the lower bounds in Theorem 2 becomes

$$(D_T)^2 = \left(\beta^T\right)^2 \left(r_A\sigma_A^2(\overline{d^T})^2 + n^{-(\delta-1)}r_\theta\sigma_\theta^2 V^{\text{in},T}\right),$$

and the rest remains the same. In this case, *ceteris paribus*, a larger n directly amplifies (if $\delta < 1$) or dampens (if $\delta > 1$) the aggregate impact of micro signals with all horizons. This means that, as $n \to \infty$ (e.g. data become more disaggregated), if the volatility of the expenditure share on each more narrowly-defined product increases $\delta \geq 0$ or decreases slowly $\delta \in (0,1)$, then micro signals could cause *more* aggregate fluctuations automatically.

It is worth emphasizing that, unlike *unweighted* indegrees, for example the firm-level connections studied by Atalay et al. (2011), *weighted* indegrees in input-output networks defined here are always finite. The following proposition ensures that the key statistics $\overline{d^T}$ and $V^{\text{in},T}$ exist and are finite.

Proposition 1. *For any order T, $\overline{d^T} \leq (\alpha_{\max})^T < 1-\gamma$ and $V^{in,T} < \frac{n}{4(n-1)}$.*

Proof. As shown in the proof of Theorem 1 in A.1.2, $\boldsymbol{d}^{\text{in},1} = \boldsymbol{\alpha} \leq \alpha_{\max}\mathbf{1}$, and so $\boldsymbol{d}^{\text{in},T} = (\operatorname{diag}(\boldsymbol{\alpha})\mathbf{\Omega})^T\mathbf{1} = (\operatorname{diag}(\boldsymbol{\alpha})\mathbf{\Omega})^{T-1}(\operatorname{diag}(\boldsymbol{\alpha})\mathbf{\Omega}\mathbf{1}) \leq \alpha_{\max}(\operatorname{diag}(\boldsymbol{\alpha})\mathbf{\Omega})^{T-1}\mathbf{1} = \alpha_{\max}\boldsymbol{d}^{\text{in},T-1} \leq \dots \leq (\alpha_{\max})^T\mathbf{1}$. The upper bound of $\overline{d^T}$ follows immediately. Meanwhile, as $d_i^{\text{in},T} < 1$ for any industry i, we have $\left(d_i^{\text{in},T}\right)^2 < d_i^{\text{in},T}, \forall\, i$. Consequently, $V^{\text{in},T} = \frac{1}{n-1}\left(\sum_i \left(d_i^{\text{in},T}\right)^2 - n(\overline{d^T})^2\right) < \frac{1}{n-1}\left(\sum_i d_i^{\text{in},T} - n(\overline{d^T})^2\right) = \frac{n}{n-1}\overline{d^T}(1-\overline{d^T}) \leq \frac{n}{4(n-1)}$. □

I require the serial dependence in $\{A_t\}$ and $\{\boldsymbol{\theta}_t\}$ to be non-negative. This ensures that the results in Theorem 2 are indeed lower bounds, achieved when $\{A_t\}$ and $\{\boldsymbol{\theta}_t\}$ are i.i.d. For example, $\ln A_t$ can be an AR(1) process with an non-negative autocorrelation coefficient ensuring $\frac{\mathrm{d}A_{t+T+\tau|t}}{\mathrm{d}A_{t+T|t}} > 0$ for any $\tau > 0$; similarly, a cross-sectionally symmetric $\boldsymbol{\theta}_t$ can be written element-wise AR(1)'s with correlated error terms, such that $\boldsymbol{\theta}_t = \rho^\theta\boldsymbol{\theta}_{t-1} + (1-\rho^\theta)\frac{1}{n} + \boldsymbol{\epsilon}_t^\theta$ with $\rho^\theta \in [0,1)$, $\mathbb{E}[\boldsymbol{\epsilon}_t^\theta] = \mathbf{0}$, $\mathbb{E}[\boldsymbol{\epsilon}_{t_1}^\theta(\boldsymbol{\epsilon}_{t_2}^\theta)'] = \mathbf{0}_{n\times n}$, and $\operatorname{corr}(\epsilon_{i,t}^\theta, \epsilon_{j,t}^\theta) = -\frac{1}{n-1}$. This implies that $\frac{\mathrm{d}\boldsymbol{\theta}_{t+T+\tau|t}}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}} \geq \mathbf{0}_{n\times n}$.

When the economy is not at its SS, $\boldsymbol{s}_t$ has the generic form in equation (2). Therefore, with respect to posterior expectations $A_{t+T|t}$ and $\boldsymbol{\theta}_{t+T|t}$,

$$\frac{\mathrm{d}\boldsymbol{s}_t}{\mathrm{d}A_{t+T|t}} \geq \frac{1}{A_t}(\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^T\boldsymbol{\theta}_{t+T|t} \geq \mathbf{0},$$
$$\frac{\mathrm{d}\boldsymbol{s}_t}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}} \geq \frac{1}{A_t}(\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^T A_{t+T|t} \geq \mathbf{0}_{n\times n}.$$

The aforementioned non-negative serial dependence ensures the first inequality in each expression, which holds at equality when the corresponding $\{A_t\}$ or $\{\boldsymbol{\theta}_t\}$ series is serially independent. Therefore, evaluated at the steady state,

$$\left.\frac{\mathrm{d}\boldsymbol{s}_t}{\mathrm{d}A_{t+T|t}}\right|_{SS} \geq \frac{1}{A}(\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^T\boldsymbol{\theta} \quad \text{and} \quad \left.\frac{\mathrm{d}\boldsymbol{s}_t}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS} \geq (\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^T.$$

Note that $\widehat{c}_t$, the first-order approximation of $\ln C_t$ around a steady state level $\ln C$ (details below), has the following form,

$$\ln\frac{C_t}{C} \approx \widehat{c}_t \equiv \left.\frac{\mathrm{d}\ln C_t}{\mathrm{d}A_{t+T|t}}\right|_{SS}(A_{t+T|t}-A) + \left.\frac{\mathrm{d}\ln C_t}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS}(\boldsymbol{\theta}_{t+T|t}-\boldsymbol{\theta}),$$

and $\widehat{q}_t$, $\widehat{h}_t$ are similar. Plugging the law of motion of $\ln \boldsymbol{y}_t$ in equation (3) into the expression of $\ln C_t$ yields

$$\ln C_t = -(1-\gamma)\boldsymbol{\theta}'\ln \boldsymbol{s}_t + \Psi_C^{En}(t-1) + \Psi_C^{Ex}(t)$$

where $\Psi^{En}(t-1)$ contains endogenous variables determined in period $t-1$ and $\Psi^{Ex}(t)$ summarizes the exogenous shocks in period t other than the signals. Neither item is affected by changes in posterior expectations $A_{t+T|t}$ and $\boldsymbol{\theta}_{t+T|t}$ caused by period-t signals about $t+T$. Therefore, if the economy in $t-1$ is at its steady state and the only shocks in period t are signals that result in $A_{t+T|t}$ and $\boldsymbol{\theta}_{t+T|t}$, the first-order derivatives of $\ln C_t$ with respect to these posterior expectations evaluated at SS are

$$\left|\left.\frac{\mathrm{d}\ln C_t}{\mathrm{d}A_{t+T|t}}\right|_{SS}\right| = \left|-(1-\gamma)\boldsymbol{\theta}'\operatorname{diag}(\boldsymbol{s})^{-1}\left.\frac{\mathrm{d}\boldsymbol{s}_t}{\mathrm{d}A_{t+T|t}}\right|_{SS}\right| \geq \frac{1-\gamma}{A}[(\boldsymbol{d}^{\text{out, all}})^{-1}]'(\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^T\boldsymbol{\theta},$$
$$\left|\left.\frac{\mathrm{d}\ln C_t}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS}\right| = \left|-(1-\gamma)\boldsymbol{\theta}'\operatorname{diag}(\boldsymbol{s})^{-1}\left.\frac{\mathrm{d}\boldsymbol{s}_t}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS}\right| \geq (1-\gamma)[(\boldsymbol{d}^{\text{out, all}})^{-1}]'(\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^T,$$

where the inverse operator (power -1) of a vector is applied element-wise, such that $\boldsymbol{x}^{-1} = (x_i{}^{-1})$, with some abuse of notation. Observe that $(\boldsymbol{d}^{\text{out, all}})^{-1} \leq (d_{\max}^{\text{out, all}})^{-1}\mathbf{1}$, which implies that

$$\left|\left.\frac{\mathrm{d}\ln C_t}{\mathrm{d}A_{t+T|t}}\right|_{SS}\right| \geq \frac{\beta^T(1-\gamma)}{Ad_{\max}^{\text{out, all}}}\frac{1}{n}\mathbf{1}'(\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^T\mathbf{1} = \frac{\beta^T(1-\gamma)}{Ad_{\max}^{\text{out, all}}}\overline{d^T},$$

$$\left|\left.\frac{\mathrm{d}\ln C_t}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS}\right| \geq \frac{\beta^T(1-\gamma)}{d^{\text{out, all}}_{\max}}\left[(\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega})^T\mathbf{1}\right]' = \frac{\beta^T(1-\gamma)}{d^{\text{out, all}}_{\max}}(\boldsymbol{d}^{\text{in},T})',$$

where the first line uses the definition of $\overline{d^T}$ such that $\overline{d^T} = \frac{1}{n}\mathbf{1}'\boldsymbol{d}^{\text{in},T} = \frac{1}{n}(\boldsymbol{d}^{\text{out},T})'\mathbf{1} = \frac{1}{n}\mathbf{1}'\boldsymbol{d}^{\text{out},T}$.

In addition, recall that the symmetry assumption of $\boldsymbol{\theta}_{t+T|t}$ requires a uniform negative cross-sectional correlation $-\frac{1}{n-1}$. This implies that, for any vector $\boldsymbol{x} \in \mathbb{R}^n$, we have

$$\begin{aligned}\boldsymbol{x}'\operatorname{var}(\boldsymbol{\theta}_{t+T|t})\boldsymbol{x} =& \frac{r_\theta\sigma_\theta^2}{n^\delta}\left(\sum_i x_i^2 - \frac{1}{n-1}\sum_i\sum_{j\neq i}x_ix_j\right)\\ =& \frac{r_\theta\sigma_\theta^2}{n^{\delta-1}}\frac{1}{n-1}\left(\sum_i x_i^2 - \frac{1}{n}\sum_i\sum_j x_ix_j\right) = \frac{r_\theta\sigma_\theta^2}{n^{\delta-1}}V^x,\end{aligned}$$

where $V^x = \frac{1}{n-1}\left(\sum_i x_i^2 - n\overline{x}^2\right)$ is the empirical cross-sectional variance in $\boldsymbol{x}$. Then, using the first-order approximation $\widehat{c}_t$ of $\ln\frac{C_t}{C}$ around SS and the conditional independence between fundamentals,

$$\begin{aligned}\operatorname{var}(\widehat{c}_t) \geq& \frac{[\beta^T(1-\gamma)]^2}{(d^{\text{out, all}}_{\max})^2}\left(\frac{(\overline{d^T})^2}{A^2}\operatorname{var}(A_{t+T|t}) + (\boldsymbol{d}^{\text{in},T})'\operatorname{var}(\boldsymbol{\theta}_{t+T|t})(\boldsymbol{d}^{\text{in},T})\right)\\ =& \frac{[\beta^T(1-\gamma)]^2}{(d^{\text{out, all}}_{\max})^2}\left(r_A\sigma_A^2(\overline{d^T})^2 + n^{-(\delta-1)}r_\theta\sigma_\theta^2V^{\text{in},T}\right).\end{aligned}$$

The variance in $\widehat{h}_t$ is much simpler, as $\ln H_t = \ln(\mathbf{1}'\boldsymbol{s}_t) + \ln A_t + \ln\gamma$. Observe that $\mathbf{1}'\boldsymbol{s}' = \frac{1}{n}\mathbf{1}'\boldsymbol{d}^{\text{out, all}} = \overline{d^{\text{all}}}$. Therefore,

$$\begin{aligned}\left.\frac{\mathrm{d}\ln H_t}{\mathrm{d}A_{t+T|t}}\right|_{SS} \geq& \frac{1}{\mathbf{1}'\boldsymbol{s}}\mathbf{1}'(\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^T\boldsymbol{\theta} = \frac{\beta^T}{\overline{d^{\text{all}}}}\overline{d^T},\\ \left.\frac{\mathrm{d}\ln H_t}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS} \geq& \frac{1}{\mathbf{1}'\boldsymbol{s}}\mathbf{1}'(\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^T = \frac{\beta^T}{\overline{d^{\text{all}}}}(\boldsymbol{d}^{\text{in},T})'.\end{aligned}$$

The volatility lower bound for labor follows.

Now we turn to the variance in $\widehat{q}_t$. When the economy in period $t-1$ is at SS, $\boldsymbol{\theta}_{t-1} = \boldsymbol{\theta}$, $\boldsymbol{s}_{t-1} = \boldsymbol{s}$ and $\boldsymbol{y}_{t-1} = \boldsymbol{y}$. Also observe that

$$\begin{aligned}(\boldsymbol{s}_t \oslash \boldsymbol{s}_{t-1}) \oslash (\boldsymbol{y}_t \oslash \boldsymbol{y}_{t-1}) =& \exp(\Delta\ln\boldsymbol{s}_t - \Delta\ln\boldsymbol{y}_t)\\ =& \exp\left((1-\gamma)\Delta\ln\boldsymbol{s}_t + \Psi_P^{En}(t-1,t-2) + \Psi_P^{Ex}(t,t-1)\right)\end{aligned}$$

where $\Psi_P^E(t-1,t-2)$ contains endogenous determined in earlier periods that are relevant for input price adjustment, and $\Psi_P^X(t,t-1)$ contains exogenous shocks other than signals. Neither is

a function of $A_{t+T|t}$ and $\boldsymbol{\theta}_{t+T|t}$. Then we have

$$\begin{aligned}\left.\frac{\mathrm{d}\ln Q_t}{\mathrm{d}\boldsymbol{s}_t}\right|_{SS} =& \left.\frac{\mathrm{d}\ln C_t}{\mathrm{d}\boldsymbol{s}_t}\right|_{SS} + 1 \times \left.\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{s}_t}\left(\mathbf{1}'\boldsymbol{s}_t - (\boldsymbol{s}-\boldsymbol{\theta})'(\boldsymbol{s}_t \oslash \boldsymbol{s}) \oslash (\boldsymbol{y}_t \oslash \boldsymbol{y})\right)\right|_{SS} \\ =& -(1-\gamma)\boldsymbol{\theta}' \operatorname{diag}(\boldsymbol{s})^{-1} + \mathbf{1}' - (\boldsymbol{s}-\boldsymbol{\theta})'\boldsymbol{I}(1-\gamma)\operatorname{diag}(\boldsymbol{s})^{-1} = \gamma\mathbf{1}'.\end{aligned}$$

Consequently,

$$\begin{aligned}\left.\frac{\mathrm{d}\ln Q_t}{\mathrm{d}A_{t+T|t}}\right|_{SS} =& \left.\frac{\mathrm{d}\ln Q_t}{\mathrm{d}\boldsymbol{s}_t}\frac{\mathrm{d}\boldsymbol{s}_t}{\mathrm{d}A_{t+T|t}}\right|_{SS} \geq \frac{\beta^T\gamma}{A}\overline{d^T}, \\ \left.\frac{\mathrm{d}\ln Q_t}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS} =& \left.\frac{\mathrm{d}\ln Q_t}{\mathrm{d}\boldsymbol{s}_t}\frac{\mathrm{d}\boldsymbol{s}_t}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS} \geq \beta^T\gamma(\boldsymbol{d}^{\text{in},T})'.\end{aligned}$$

The volatility lower bound for real GDP follows.

A.2.3 Post period t dynamics: physical and belief-based transmission channels

A full characterization of the dynamics of $\widehat{c_{t+\tau}}, \widehat{h_{t+\tau}}, \widehat{q_{t+\tau}}$ is feasible but lengthy, precisely because of the coexistence of the two intertemporal transmission channels discussed in section 4.1: a physical input-flow channel and a belief-based channel. Therefore, I will discuss the main ingredients of the two channels but omit the detailed characterization. Recall that the dynamics are captured by $\boldsymbol{s}_{t+\tau}$ and $\ln \boldsymbol{y}_{t+\tau}$.

To illustrate the pure physical channel of input flows, it is without loss to consider the transmission of shocks to $\ln \boldsymbol{z}_t$. As stated before, all realized shocks share the physical channel. Moreover, $\ln \boldsymbol{z}_t$ does not affect agents' beliefs or $\boldsymbol{s}_{t+\tau}$. Then, the one-period transmission of productivity shocks at the industry level satisfies

$$\frac{\mathrm{d}\ln \boldsymbol{y}_{t+1}}{\mathrm{d}\ln \boldsymbol{z}_t} = \operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}\frac{\mathrm{d}\ln \boldsymbol{y}_t}{\mathrm{d}\ln \boldsymbol{z}_t} = \operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}.$$

In the aggregate, the strength of the physical channel of input flows is captured by

$$\left.\frac{\mathrm{d}\ln C_{t+1}}{\mathrm{d}\ln \boldsymbol{z}_t}\right|_{SS} = \boldsymbol{\theta}'\frac{\mathrm{d}\ln \boldsymbol{y}_{t+1}}{\mathrm{d}\ln \boldsymbol{z}_t} = \frac{1}{n}\mathbf{1}'\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega} = \frac{1}{n}(\boldsymbol{d}^{\text{out},1})',$$

which sets the magnitude of $\widehat{c_{t+1}}$ and $\widehat{q_{t+1}}$. Analogously, over multiple periods, $\left.\frac{\mathrm{d}\ln C_{t+\tau}}{\mathrm{d}\ln \boldsymbol{z}_t}\right|_{SS} = \frac{1}{n}(\boldsymbol{d}^{\text{out},\tau})'$.

The belief-based channel for intertemporal transmission is more complicated, as a consequence of input time to build. This is not only because signals have persistent impacts on agent decisions due to signals' persistent impacts on beliefs about relevant fundamentals, but also because these decisions affect future equilibrium outcome through the physical input-flow channel as discussed above. I focus on the first aspect which is the persistent impact of earlier signals on beliefs and hence decisions. The belief-based channel builds on the following blocks. Affecting all allocations

including labor and final use, when $\tau \leq T-1$,

$$\left.\frac{\mathrm{d}\boldsymbol{s}_{t+\tau}}{\mathrm{d}A_{t+T|t}}\right|_{SS} \geq \frac{1}{A}(\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^{T-\tau}\boldsymbol{\theta}\frac{\mathrm{d}A_{t+T|t+\tau}}{\mathrm{d}A_{t+T|t}} \geq \mathbf{0},$$
$$\left.\frac{\mathrm{d}\boldsymbol{s}_{t+\tau}}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS} \geq (\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^{T-\tau}\frac{\mathrm{d}\boldsymbol{\theta}_{t+T|t+\tau}}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}} \geq \mathbf{0}_{n\times n}.$$

The persistent impact of period-t signals on period-$(t+\tau)$ beliefs are captured by $\frac{\mathrm{d}A_{t+T|t+\tau}}{\mathrm{d}A_{t+T|t}}$ and $\frac{\mathrm{d}\boldsymbol{\theta}_{t+T|t+\tau}}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}$. They are non-negative and take the value of zeros only if the fundamentals A_{t+T} and $\boldsymbol{\theta}_{t+T}$ are fully revealed at $t+\tau$; in the other extreme, if no additional signals arrive after t, then $\frac{\mathrm{d}A_{t+T|t+\tau}}{\mathrm{d}A_{t+T|t}} = 1$ and $\frac{\mathrm{d}\boldsymbol{\theta}_{t+T|t+\tau}}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}} = \boldsymbol{I}$. The latter is the case for the experiments in section 4.2. Without loss of generality, let $r_{A,\tau} > 0$ and $r_{\theta,\tau} > 0$ capture the minimum persistence in beliefs, which decreases in signal precisions. Then,

$$\left.\frac{\mathrm{d}\boldsymbol{s}_{t+\tau}}{\mathrm{d}A_{t+T|t}}\right|_{SS} \geq \frac{1}{A}(\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^{T-\tau}\boldsymbol{\theta}\, r_{A,\tau} = \frac{r_{A,\tau}\beta^{T-\tau}}{An}(\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^{T-\tau}\mathbf{1},$$
$$\left.\frac{\mathrm{d}\boldsymbol{s}_{t+\tau}}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS} \geq (\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^{T-\tau} r_{\theta,\tau} = r_{\theta,\tau}\beta^{T-\tau}(\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^{T-\tau}.$$

In the aggregate, for example, the magnitude of $\widehat{h_{t+\tau}}$ is then determined by

$$\left.\frac{\mathrm{d}\ln H_{t+\tau}}{\mathrm{d}A_{t+T|t}}\right|_{SS} \geq \frac{r_{A,\tau}\beta^{T-\tau}}{An\mathbf{1}'\boldsymbol{s}}\mathbf{1}'(\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^{T-\tau}\mathbf{1} = \frac{r_{A,\tau}\beta^{T-\tau}}{A\overline{d^{\text{all}}}}\overline{d^{T-\tau}},$$
$$\left.\frac{\mathrm{d}\ln H_{t+\tau}}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS} \geq \frac{r_{\theta,\tau}\beta^{T-\tau}}{\mathbf{1}'\boldsymbol{s}}\mathbf{1}'(\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^{T-\tau} = \frac{r_{\theta,\tau}\beta^{T-\tau}}{\overline{d^{\text{all}}}}(\boldsymbol{d}^{\text{in},T-\tau})'.$$

Note that, as is the case of section 4.2, when belief updating is sufficiently sluggish ($r_{A,\tau}$ is close or equal to 1), $\widehat{h_{t+\tau}}$ is monotone in τ for $\tau = 0,\ldots,T-1$ in response to macro signals. This is because degree average $\overline{d^{\tau}}$ decreases in order τ. The same force contributes to the dynamics of other aggregate variables. Whether $\widehat{h_{t+\tau}}$ is monotone in τ in response to micro signals depends on $r_{\theta,\tau}$ but also on the distribution of indegrees as well as the actual signal realization. The allocation of inputs in period $t+\tau$ is additionally determined by $\mathbb{E}_{t+\tau}[A_{t+\tau+1}\boldsymbol{s}_{t+\tau+1}]/A_{t+\tau}$. The analysis is similar and thus omitted.

A.3 Details of Simple Examples

Each economy starts at its deterministic steady state. In period $t_0 = 0$, $\mathcal{M}_{t_0}$ arrives, containing horizon-T signals about A_{t+T} and/or $\boldsymbol{\theta}_{t+T}$ with $T = 4$. All other sources of fluctuation are shut down. Agents know that $A_t = A$ and $\boldsymbol{\theta}_t = \frac{1}{n}\mathbf{1}$, $\forall t \neq t_0 + T$. Agents do not receive additional information in any other periods $t \neq t_0$. This means that, after observing $\mathcal{M}_{t_0}$, agents' beliefs about $t_0 + T$ remain unchanged in every period from t_0 to $t_0 + T - 1$.

In the macro signal only scenario, $A_{t_0+T|t_0} = A_{t_0+T|t_0+1} = \ldots = (1+\Delta)A$, $\Delta > 0$. In the micro

signal only scenario, $\boldsymbol{\theta}_{t_0+T|t_0} = \boldsymbol{\theta}_{t_0+T|t_0+1} = \ldots = \frac{1}{n(1+\Delta)}\mathbf{1} + \frac{\Delta}{1+\Delta}\boldsymbol{e}_i$, with $\boldsymbol{e}_i$ being the i-th standard basis of $\mathbb{R}^n$.

The common parameters for the simulation of each example are as follows. The discount factor is $\beta = 0.99$. The aggregate preference process has mean $A = 1$. The mean of final-use distribution is $\boldsymbol{\theta} = \frac{1}{n}\mathbf{1}$. The labor input share in each industry is $\gamma = 0.5$. Productivity is fixed at $\boldsymbol{z} = \mathbf{1}$. The size of shocks is $\Delta = 0.01$.

The first set of economies are finite production chains. The direction of input flow is from j to $j+1$, $\forall j$, $1 \leq j < n$. Given n, the input-output matrix in such an economy is

$$[\mathrm{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}]^{\text{chain}} = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 \\ \alpha_2 & 0 & \ldots & 0 & 0 \\ 0 & \alpha_3 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & \alpha_n & 0 \end{bmatrix}.$$

The four chains differ in $\boldsymbol{\alpha}$. Note that $\boldsymbol{d}^{\text{out},\tau} = \boldsymbol{d}^{\text{in},\tau} = \mathbf{0}$ for any $\tau \geq n$ regardless of $\boldsymbol{\alpha}$. For the light chain, all industries share $\alpha^{\text{light}} = 0.1$; for the heavy chain, all industries share $\alpha^{\text{heavy}} = 0.5$. For the chain with increasing input shares, $\alpha_j^{\text{increase}}$ increases linearly from $\alpha_2^{\text{increase}} = 0.1$ to $\alpha_n^{\text{increase}} = 0.5$. Similarly, $\alpha_j^{\text{decrease}}$ decreases linearly from $\alpha_2^{\text{decrease}} = 0.5$ to $\alpha_n^{\text{decrease}} = 0.1$.

The second set of economies are symmetric, with a common input share $\alpha = 0.5$. For a given n, the cross-industry connections are summarized by $\boldsymbol{\Omega}$ as follows

$$\boldsymbol{\Omega}^{\text{scattered}} = \boldsymbol{I}, \quad \boldsymbol{\Omega}^{\text{ring}} = \begin{bmatrix} 0 & 0 & \ldots & 0 & 1 \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix}, \quad \boldsymbol{\Omega}^{\text{complete}} = \frac{1}{n}\mathbf{1}_{n\times n}.$$

In all three cases, $V^{\text{in},\tau} = 0$ for any order $\tau \geq 1$ as $\boldsymbol{d}^{\text{in},\tau} = \boldsymbol{d}^{\text{out},\tau} = \alpha^\tau \mathbf{1}$.

A.4 Details of Section 4.3 and Proof of Theorem 3

Without product-specific preference shifts, equation (2) becomes

$$\boldsymbol{s}_t = \boldsymbol{\theta} + \frac{1}{A_t}\sum_{\tau=1}^{\infty} A_{t+\tau|t}\left[\beta\boldsymbol{\Omega}'\,\mathrm{diag}(\boldsymbol{\alpha})\right]^\tau \boldsymbol{\theta}.$$

To ease expression, I introduce period-t forecast vector such that $\boldsymbol{f}_t = \mathbb{E}_t[A_{t+1}\boldsymbol{s}_{t+1}]/A_t$. In this case, without micro-level shocks,

$$\boldsymbol{f}_t = \frac{1}{A_t}\sum_{\tau=1}^{\infty} A_{t+\tau|t}\left[\beta\boldsymbol{\Omega}'\,\mathrm{diag}(\boldsymbol{\alpha})\right]^{\tau-1} \boldsymbol{\theta}.$$

The rest of the equilibrium outcome then follows, including $h_t = A_t \boldsymbol{\gamma}' \boldsymbol{s}_t$. In particular, we can write the law of motion of aggregate final use growth $\Delta \ln C_t = \sum_i \theta_i \Delta \ln c_{i,t}$. First, note that

$$c_{i,t} = y_{i,t} \frac{\theta_i}{s_{i,t}} \implies \ln c_{i,t} = \ln y_{i,t} - \ln s_{i,t} + \ln \theta_i.$$

Then, without product-specific preference shifts, equation (3) remains

$$\ln \boldsymbol{y}_t = \operatorname{diag}(\boldsymbol{\alpha}) \boldsymbol{\Omega} \ln \boldsymbol{y}_{t-1} + \ln \boldsymbol{z}_t + \boldsymbol{\gamma} \ln A_t + \operatorname{diag}(\boldsymbol{\gamma}) \ln \boldsymbol{s}_t + \operatorname{diag}(\boldsymbol{\alpha}) \left(\ln \boldsymbol{f}_{t-1} - \boldsymbol{\Omega} \ln \boldsymbol{s}_{t-1} \right) + \boldsymbol{\psi}.$$

Replacing $\ln \boldsymbol{y}_t$ with $\ln \boldsymbol{c}_t$ yields

$$\begin{aligned} \ln \boldsymbol{c}_t + \ln \boldsymbol{s}_t =& \operatorname{diag}(\boldsymbol{\alpha}) \boldsymbol{\Omega} [\ln \boldsymbol{c}_{t-1} + \ln \boldsymbol{s}_{t-1}] + \ln \boldsymbol{z}_t + \boldsymbol{\gamma} \ln A_t \\ &+ \operatorname{diag}(\boldsymbol{\gamma}) \ln \boldsymbol{s}_t + \operatorname{diag}(\boldsymbol{\alpha}) \left(\ln \boldsymbol{f}_{t-1} - \boldsymbol{\Omega} \ln \boldsymbol{s}_{t-1} \right) + \widetilde{\boldsymbol{\psi}} \\ =& \operatorname{diag}(\boldsymbol{\alpha}) \boldsymbol{\Omega} \ln \boldsymbol{c}_{t-1} + \ln \boldsymbol{z}_t + \boldsymbol{\gamma} \ln A_t + \operatorname{diag}(\boldsymbol{\gamma}) \ln \boldsymbol{s}_t + \operatorname{diag}(\boldsymbol{\alpha}) \ln \boldsymbol{f}_{t-1} + \widetilde{\boldsymbol{\psi}} \end{aligned}$$

where $\widetilde{\boldsymbol{\psi}} = \boldsymbol{\psi} + [\boldsymbol{I} - \operatorname{diag}(\boldsymbol{\alpha}) \boldsymbol{\Omega}] \ln \boldsymbol{\theta}$. Rearranging and left-multiplying both sides by $\boldsymbol{\theta}'$ produces

$$\begin{aligned} \ln C_t = \sum \theta_i \ln c_{i,t} =& \boldsymbol{\theta}' \operatorname{diag}(\boldsymbol{\alpha}) \boldsymbol{\Omega} \ln \boldsymbol{c}_{t-1} + \boldsymbol{\theta}' \ln \boldsymbol{z}_t + \boldsymbol{\theta}' \boldsymbol{\gamma} \ln A_t \\ &- \sum \theta_i (1 - \gamma_i) \ln s_{i,t} + \sum \theta_i \alpha_i \ln f_{i,t-1} + \boldsymbol{\theta}' \widetilde{\boldsymbol{\psi}}. \end{aligned} \tag{10}$$

Observe that for an arbitrary forecast horizon $T \geq 1$, to ensure irrelevance we need to eliminate $\boldsymbol{\Omega}$ from the expressions for all aggregate-level variables. This requires $\boldsymbol{\Omega}$ to disappear from $\boldsymbol{s}_t$. Hence we need $\widetilde{\alpha} \in (0, 1)$ such that

$$\boldsymbol{\Omega}' \operatorname{diag}(\boldsymbol{\alpha}) \boldsymbol{\theta} = \widetilde{\alpha} \boldsymbol{\theta},$$

or $\widetilde{\alpha} \in (0, 1)$ is one of the eigenvalues of $\boldsymbol{\Omega}' \operatorname{diag}(\boldsymbol{\alpha})$ with $\boldsymbol{\theta}$ being the associated eigenvector. The tighter upper bound $\widetilde{\alpha} \leq \alpha_{\max}$ results from the Perron-Frobenius theorem. Eigenvalues of a non-negative matrix are bounded above by the largest row sum. Under this condition,

$$\boldsymbol{\theta}' \operatorname{diag}(\boldsymbol{\alpha}) \boldsymbol{\Omega} \ln \boldsymbol{c}_{t-1} = (\boldsymbol{\Omega}' \operatorname{diag}(\boldsymbol{\alpha}) \boldsymbol{\theta})' \ln \boldsymbol{c}_{t-1} = (\widetilde{\alpha} \boldsymbol{\theta})' \ln \boldsymbol{c}_{t-1} = \widetilde{\alpha} \ln C_{t-1},$$

and

$$\boldsymbol{s}_t = \frac{1}{A_t} \left[A_t + \sum_{\tau=1}^{\infty} (\beta \widetilde{\alpha})^{\tau} A_{t+\tau|t} \right] \boldsymbol{\theta} \equiv \widetilde{s}_t \boldsymbol{\theta}, \quad \boldsymbol{f}_t = \frac{1}{A_t} \left[\sum_{\tau=1}^{\infty} (\beta \widetilde{\alpha})^{\tau-1} A_{t+\tau|t} \right] \boldsymbol{\theta} \equiv \widetilde{f}_t \boldsymbol{\theta}.$$

Define $E_t(\widetilde{\alpha}) \equiv -(1 - \boldsymbol{\theta}' \boldsymbol{\gamma}) \ln \widetilde{s}_t + \boldsymbol{\theta}' \boldsymbol{\alpha} \ln \widetilde{f_{t-1}}$. Then $E_t(\widetilde{\alpha})$ summarizes all relevant posterior expectations $\{A_{t-1+\tau|t-1}, A_{t+\tau|t}\}_{\tau \geq 1}$ and realized A_{t-1}, A_t, and $E_t(\widetilde{\alpha})$ depends on the production technologies including the network structure through $\widetilde{\alpha}$ and $\boldsymbol{\theta}' \boldsymbol{\alpha}, \boldsymbol{\theta}' \boldsymbol{\gamma}$. Equation (10) becomes equation (5) if $z_{i,t} = z_t$ for all firms, i.e.,

$$\ln C_t = \widetilde{\alpha} \ln C_{t-1} + \ln z_t + \boldsymbol{\theta}' \boldsymbol{\gamma} \ln A_t + E_t(\widetilde{\alpha}) + \Psi^C,$$

where $\Psi^C = \boldsymbol{\theta}'\widetilde{\boldsymbol{\psi}} - \sum \theta_i(1-\alpha_i-\gamma_i)\ln\theta_i$. Meanwhile, the aggregate labor supply satisfies $h_t = A_t\boldsymbol{\gamma}'\boldsymbol{s}_t = \boldsymbol{\theta}'\boldsymbol{\gamma}\left[A_t + \sum_{\tau=1}^{\infty}(\beta\widetilde{\alpha})^\tau A_{t+\tau|t}\right]$.

Under the irrelevance condition, the aggregate real value added or Q_t satisfies

$$\begin{aligned}\ln Q_t =& \ln C_t + \ln\left(\mathbf{1}'\boldsymbol{s}_t - (\boldsymbol{s}_{t-1}-\boldsymbol{\theta})'[(\boldsymbol{s}_t \oslash \boldsymbol{s}_{t-1}) \oslash (\boldsymbol{y}_t \oslash \boldsymbol{y}_{t-1})]\right)\\ =& \ln C_t + \ln\left(\widetilde{s}_t - (\widetilde{s_{t-1}}-1)\boldsymbol{\theta}'\exp(-\Delta\ln\boldsymbol{c}_t)\right),\end{aligned}$$

which is because $\Delta\ln\boldsymbol{s}_t - \Delta\ln\boldsymbol{y}_t = -\Delta\ln\boldsymbol{c}_t$ when $\boldsymbol{\theta}_t = \boldsymbol{\theta}$, $\forall t$. As a first-order approximation around $\ln\boldsymbol{c}_{t-1}$, we have

$$\exp(-\Delta\ln\boldsymbol{c}_t) \approx \mathbf{1} + \left.\frac{\mathrm{d}\exp(\ln\boldsymbol{c}_{t-1}-\ln\boldsymbol{c}_t)}{\mathrm{d}\ln\boldsymbol{c}_t}\right|_{\boldsymbol{c}_t=\boldsymbol{c}_{t-1}} \Delta\ln\boldsymbol{c}_t = \mathbf{1} - \Delta\ln\boldsymbol{c}_t.$$

It follows that $\ln Q_t$ is independent of the matrix $\boldsymbol{\Omega}$ up to a first-order approximation, such that

$$\begin{aligned}\ln Q_t \approx& \ln C_t + \ln\left(\widetilde{s}_t - (\widetilde{s_{t-1}}-1)\boldsymbol{\theta}'(1-\Delta\ln\boldsymbol{c}_t)\right)\\ =& \ln C_t + \ln\left(\widetilde{s}_t + (\widetilde{s_{t-1}}-1)\Delta\ln C_t - \widetilde{s_{t-1}} + 1\right) \approx \ln C_t + (\widetilde{s_{t-1}}-1)\Delta\ln C_t + \Delta\widetilde{s}_t.\end{aligned}$$

However, for the dynamics of $\ln Q_t$ to be completely independent of $\boldsymbol{\Omega}$, we need $\exp(-\Delta\ln\boldsymbol{c}_t)$ and hence $\Delta\ln\boldsymbol{c}_t$ to be independent of $\boldsymbol{\Omega}$:

$$\Delta\ln\boldsymbol{c}_t = \mathrm{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}\Delta\ln\boldsymbol{c}_{t-1} + \Delta\ln z_t + \boldsymbol{\gamma}\Delta\ln A_t - (1-\boldsymbol{\gamma})\Delta\ln\widetilde{s}_t + \boldsymbol{\alpha}\Delta\ln\widetilde{f_{t-1}}.$$

A sufficient condition is that $\Delta\ln c_{i,t} = \Delta\ln c_{j,t}$, $\forall i,j$ and $\forall t$, which calls for $\alpha_i = \alpha_j$ and $\gamma_i = \gamma_j$, $\forall i,j$, whereas $\boldsymbol{\Omega}$ can be arbitrary as long as $\sum_j \omega_{ij} = 1$, $\forall i$.

The representative-firm model with equivalent aggregate-level equilibrium outcome is as follows. The firm uses part of its own product towards future production, with a production function $y_t = \widetilde{z}_t h_t^{\widetilde{\gamma}} x_{t-1}^{\widetilde{\alpha}}$, where $\widetilde{\gamma} = \boldsymbol{\theta}'\boldsymbol{\gamma}$. The rest of the product is sold to the households for consumption. The preferences of the households have the same form, $A_t\ln C_t - h_t$. The other aspects of the model do not change. The product market-clearing condition is $y_t = C_t + x_t$. Then it is straightforward to show that, in equilibrium,

$$s_t \equiv \frac{y_t}{C_t} = 1 + \frac{\beta\widetilde{\alpha}}{A_t}\mathbb{E}_t[A_{t+1}s_{t+1}] = 1 + \frac{1}{A_t}\sum_{\tau=1}^{\infty}(\beta\widetilde{\alpha})^\tau A_{t+\tau|t}$$

and $f_t \equiv \sum_{\tau=1}^{\infty}(\beta\widetilde{\alpha})^{\tau-1}A_{t+\tau|t}/A_t$ with $x_t = C_t f_t$. Then, $h_t = A_t\widetilde{\gamma}s_t = \widetilde{\gamma}\left[A_t + \sum_{\tau=1}^{\infty}(\beta\widetilde{\alpha})^\tau A_{t+\tau|t}\right]$, and the law of motion for consumption becomes

$$\begin{aligned}\ln C_t =& \ln y_t - \ln s_t\\ =& \ln\widetilde{z}_t + \widetilde{\gamma}(\ln A_t + \ln s_t) + \widetilde{\alpha}(\ln f_{t-1} + \ln C_{t-1}) - \ln s_t\\ =& \widetilde{\alpha}\ln C_{t-1} + \ln\widetilde{z}_t + \widetilde{\gamma}\ln A_t + \widetilde{\alpha}\ln f_{t-1} - (1-\widetilde{\gamma})\ln s_t.\end{aligned}$$

Compared with equation (5) for an economy with interconnected industries, for the law of motion to be identical, we further require $\widetilde{\alpha} = \boldsymbol{\theta}'\boldsymbol{\alpha}$ and $\ln \widetilde{z}_t = \psi^C + \ln z_t$.

B Model Variants and Extensions

B.1 Heterogenous Time to Build

B.1.1 Zero- or one-period time to build

Suppose the production function of firm i becomes $y_{i,t} = z_{i,t} h_{i,t}^{\gamma_i} M_{i,t}^{\alpha_{i,0}} X_{i,t-1}^{\alpha_{i,1}}$, with $\gamma_i + \alpha_{i,0} + \alpha_{i,1} \leq 1$. Here $M_{i,t}$ is firm i's compound of just-in-time (jit) inputs purchased in period t, $M_{i,t} = \prod_j m_{ij,t}^{\omega_{ij,0}}$, where $\omega_{ij,0} \geq 0, \sum_j \omega_{ij,0} = 1$. Denote $\boldsymbol{\alpha}_0 = (\alpha_{i,0})$ and $\boldsymbol{\Omega}_0 = [\omega_{ij,0}]$ as the corresponding parts of input-output matrix associated with these inputs. Let $X_{i,t-1} = \prod_j x_{ij,t-1}^{\omega_{ij,1}}$ denote the time-to-build (ttb) input compound that still requires purchase in advance, with $\omega_{ij,1} \geq 0, \sum_j \omega_{ij,1} = 1$. Denote $\boldsymbol{\alpha}_1 = (\alpha_{i,1})$ and $\boldsymbol{\Omega}_1 = [\omega_{ij,1}]$. The household's problem remains the same.

Firm i's flow profit then becomes $\pi_{i,t} = p_{i,t} y_{i,t} - w_t h_{i,t} - \sum_j p_{j,t}(x_{ij,t} + m_{ij,t})$. Its optimal decision on jit input expenditure is static, such that $\frac{p_{j,t} m_{ij,t}}{P_t C_t} = \alpha_{i,0}\omega_{ij,0} s_{i,t}$. Unlike the ttb inputs, expenditure on jit inputs does not explicitly rely on firm i's expectation of its future sales. Firm i's expenditure on input good j in period t relative to aggregate final use therefore becomes

$$\frac{p_{j,t}(x_{ij,t} + m_{ij,t})}{P_t C_t} = \alpha_{i,0}\omega_{ij,0} s_{i,t} + \frac{1}{A_t}\beta\alpha_{i,1}\omega_{ij,1}\,\mathbb{E}_t[A_{t+1} s_{i,t+1}].$$

The market-clearing condition for each good now becomes $y_{i,t} = c_{i,t} + \sum_j m_{ji,t} + \sum_j x_{ji,t}$. Evaluating it at the equilibrium yields $\boldsymbol{s}_t = \boldsymbol{\theta}_t + \boldsymbol{\Omega}_0' \operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{s}_t + \frac{1}{A_t}\beta\boldsymbol{\Omega}_1' \operatorname{diag}(\boldsymbol{\alpha}_1)\,\mathbb{E}_t[A_{t+1}\boldsymbol{s}_{t+1}]$, or, equivalently,

$$\boldsymbol{s}_t = [\boldsymbol{I} - \boldsymbol{\Omega}_0' \operatorname{diag}(\boldsymbol{\alpha}_0)]^{-1}\left(\boldsymbol{\theta}_t + \frac{1}{A_t}\beta\boldsymbol{\Omega}_1' \operatorname{diag}(\boldsymbol{\alpha}_1)\,\mathbb{E}_t[A_{t+1}\boldsymbol{s}_{t+1}]\right).$$

The steady-state Domar weights thus become $\boldsymbol{s} = [\boldsymbol{I} - \boldsymbol{\Omega}_0' \operatorname{diag}(\boldsymbol{\alpha}_0) - \beta\boldsymbol{\Omega}_1' \operatorname{diag}(\boldsymbol{\alpha}_1)]^{-1}\boldsymbol{\theta}$. Let $\boldsymbol{L}_0$ be the Leontief inverse such that $\boldsymbol{L}_0 = [\boldsymbol{I} - \operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0]^{-1}$, then

$$\boldsymbol{s}_t = \boldsymbol{L}_0'\boldsymbol{\theta}_t + \frac{1}{A_t}\sum_{\tau=1}^{\infty}\left(\beta\boldsymbol{L}_0'\boldsymbol{\Omega}' \operatorname{diag}(\boldsymbol{\alpha})\right)^{\tau} A_{t+\tau|t}\boldsymbol{L}_0'\boldsymbol{\theta}_{t+\tau|t}.$$

Real industrial output satisfies

$$\begin{aligned}\ln \boldsymbol{y}_t =& \ln \boldsymbol{z}_t + \boldsymbol{\gamma}\ln A_t + \operatorname{diag}(\boldsymbol{\gamma})\ln \boldsymbol{s}_t + \operatorname{diag}(\boldsymbol{\alpha}_0)\left(\boldsymbol{\Omega}_0 \ln \boldsymbol{y}_t + \ln \boldsymbol{s}_t - \boldsymbol{\Omega}_0 \ln \boldsymbol{s}_t\right)\\ &+ \operatorname{diag}(\boldsymbol{\alpha})\left(\boldsymbol{\Omega}\ln \boldsymbol{y}_{t-1} + \ln(\mathbb{E}_{t-1}[A_t\boldsymbol{s}_t]/A_{t-1}) - \boldsymbol{\Omega}\ln \boldsymbol{s}_{t-1}\right) + \boldsymbol{\psi}\end{aligned}$$

which becomes the law of motion,

$$\ln \boldsymbol{y}_t = \boldsymbol{L}_0\big[\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}\ln \boldsymbol{y}_{t-1} + \ln \boldsymbol{z}_t + \boldsymbol{\gamma}\ln A_t + \operatorname{diag}(\boldsymbol{\gamma})\ln \boldsymbol{s}_t$$

$$+\operatorname{diag}(\boldsymbol{\alpha}_0)\left(\boldsymbol{I}-\boldsymbol{\Omega}_0\right)\ln \boldsymbol{s}_t+\operatorname{diag}(\boldsymbol{\alpha})\left(\ln(\mathbb{E}_{t-1}[A_t\boldsymbol{s}_t]/A_{t-1})-\boldsymbol{\Omega}\ln \boldsymbol{s}_{t-1}\right)+\boldsymbol{\psi}\big].$$

Aggregate real final use C_t remains unchanged, which satisfies $\ln C_t = \boldsymbol{\theta}'\ln \boldsymbol{c}_t = \boldsymbol{\theta}'(\ln \boldsymbol{y}_t + \ln \boldsymbol{\theta}_t - \ln \boldsymbol{s}_t)$. Aggregate labor H_t remains unchanged, such that $\ln H_t = \ln(\boldsymbol{\gamma}'\boldsymbol{s}_t) + \ln A_t$. Aggregate real value added Q_t needs an adjustment in the prices for ttb inputs, and so the accounting rule requires that

$$\begin{aligned}Q_t =& \sum_i \frac{p_{i,t}}{P_t}y_{i,t} - \sum_i\sum_j \frac{p_{j,t}}{P_t}(m_{ij,t}+x_{ij,t-1})\\ =& C_t\sum_i s_{i,t} - C_t\sum_i \alpha_{i,0}s_{i,t} - C_{t-1}\sum_j \frac{p_{j,t}/p_{j,t-1}}{P_t/P_{t-1}}(s_{j,t-1}-\theta_{j,t-1}-\sum_i \alpha_{i,0}\omega_{ij,0}s_{i,t-1}).\end{aligned}$$

Then, written in logarithms, it becomes

$$\ln Q_t = \ln C_t + \ln\left((\boldsymbol{1}-\boldsymbol{\alpha}_0)'\boldsymbol{s}_t - [(\boldsymbol{I}-\boldsymbol{\Omega}_0'\operatorname{diag}(\boldsymbol{\alpha}_0))\boldsymbol{s}_{t-1}-\boldsymbol{\theta}_{t-1}]'[(\boldsymbol{s}_t\oslash\boldsymbol{s}_{t-1})\oslash(\boldsymbol{y}_t\oslash\boldsymbol{y}_{t-1})]\right).$$

B.1.2 Redefined indegrees and outdegrees

Redefine the T-th order *indegree* and *outdegree* of each industry as follows

$$\widetilde{\boldsymbol{d}^{\text{in},T}} \equiv [\boldsymbol{L}_0\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1]^T\boldsymbol{1}, \qquad \widetilde{\boldsymbol{d}^{\text{out},T}} \equiv [\boldsymbol{\Omega}_1'\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{L}_0']^T\boldsymbol{1}.$$

Then we have $\widetilde{\boldsymbol{d}^{\text{out, all}}} \equiv \boldsymbol{1}+\sum_{\tau=1}^{\infty}\beta^\tau\widetilde{\boldsymbol{d}^{\text{out},\tau}}$. The equilibrium Domar weights at the deterministic steady state satisfy $n\boldsymbol{s} = \boldsymbol{L}_0'\left[\boldsymbol{1}+\sum_{\tau\geq 1}(\beta\boldsymbol{\Omega}_1'\operatorname{diag}(\boldsymbol{\alpha})\boldsymbol{L}_0')^\tau\boldsymbol{1}\right] = \boldsymbol{L}_0'\widetilde{\boldsymbol{d}^{\text{out, all}}}$. The comparison of indegrees defined with and without JIT inputs is summarized below.

Proposition 2. *For any order T, $\widetilde{\boldsymbol{d}^{in,T}} \leq \boldsymbol{d}^{in,T} \equiv (\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0+\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1)^T\boldsymbol{1}$.*

Proof. The proof is by deduction. At $\tau = 1$, need to show $\widetilde{\boldsymbol{d}^{\text{in},1}} = \boldsymbol{L}_0\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1\boldsymbol{1} = [\boldsymbol{I}-\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0]^{-1}\boldsymbol{\alpha}_1 \leq (\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0+\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1)\boldsymbol{1} = \boldsymbol{\alpha}_0+\boldsymbol{\alpha}_1 = \boldsymbol{d}^{\text{in},1}$:

$$\begin{aligned}&[\boldsymbol{I}-\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0]^{-1}\boldsymbol{\alpha}_1 \leq \boldsymbol{\alpha}_0+\boldsymbol{\alpha}_1\\ \Longleftarrow\ &\boldsymbol{\alpha}_1 \leq [\boldsymbol{I}-\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0](\boldsymbol{\alpha}_0+\boldsymbol{\alpha}_1) = \boldsymbol{\alpha}_1+\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0(\boldsymbol{1}-\boldsymbol{\alpha}_0-\boldsymbol{\alpha}_1)\\ \Longleftarrow\ &\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0(\boldsymbol{1}-\boldsymbol{\alpha}_0-\boldsymbol{\alpha}_1) \geq \operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0\boldsymbol{\gamma}\boldsymbol{1} = \boldsymbol{\gamma}\boldsymbol{\alpha}_0 \geq \boldsymbol{0},\end{aligned} \tag{11}$$

where the first " $\Longleftarrow$ " holds because both sides of (11) are element-wise non-negative and $\boldsymbol{L}_0 = [\boldsymbol{I}-\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0]^{-1}$ is also element-wise non-negative.

Now, suppose $\widetilde{\boldsymbol{d}^{\text{in},\tau}} \leq \boldsymbol{d}^{\text{in},\tau}$ for some $\tau \geq 1$, need to show $\widetilde{\boldsymbol{d}^{\text{in},\tau+1}} = \boldsymbol{L}_0\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1\widetilde{\boldsymbol{d}^{\text{in},\tau}} \leq (\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0+\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1)\boldsymbol{d}^{\text{in},\tau} = \boldsymbol{d}^{\text{in},\tau+1}$:

$$\begin{aligned}&\boldsymbol{L}_0\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1\widetilde{\boldsymbol{d}^{\text{in},\tau}} \leq (\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0+\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1)\boldsymbol{d}^{\text{in},\tau}\\ \Longleftarrow\ &\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1\widetilde{\boldsymbol{d}^{\text{in},\tau}} \leq [\boldsymbol{I}-\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0](\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0+\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1)\boldsymbol{d}^{\text{in},\tau}\end{aligned}$$

$$\begin{aligned}
&\Longleftarrow \operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1(\widetilde{\boldsymbol{d}^{\text{in},\tau}} - \boldsymbol{d}^{\text{in},\tau}) \leq \operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0[\boldsymbol{I} - \operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0 - \operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1]\boldsymbol{d}^{\text{in},\tau} \\
&\Longleftarrow \operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{\Omega}_1(\boldsymbol{d}^{\text{in},\tau} - \widetilde{\boldsymbol{d}^{\text{in},\tau}}) + \operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0(\boldsymbol{d}^{\text{in},\tau} - \boldsymbol{d}^{\text{in},\tau+1}) \geq \boldsymbol{0},
\end{aligned}$$

where the first " $\Longleftarrow$ " holds for the same reason, and the last inequality holds because $\widetilde{\boldsymbol{d}^{\text{in},\tau}} \leq \boldsymbol{d}^{\text{in},\tau}$ as assumed and $\boldsymbol{d}^{\text{in},\tau} \geq (\alpha_{0,\max} + \alpha_{1,\max})\boldsymbol{d}^{\text{in},\tau} \geq \boldsymbol{d}^{\text{in},\tau+1}$ by definition of indegrees.

□

This proposition thus ensures that $\widetilde{\overline{d^T}} \leq \overline{d^T}$ at any order T. But it does not guarantee the same for variances $\widetilde{V^{\text{in},T}}$ and $V^{\text{in},T}$.

B.1.3 Counterpart of Theorem 2 with JIT inputs

As in the proof of Theorem 2 in A.2, I consider the more general case with $\text{var}(\theta_{i,t}) = n^{-\delta}\sigma_\theta^2$, for some δ. The counterpart of Theorem 2 with JIT inputs is as follows.

Theorem 4. *(Volatility Lower Bound with JIT Inputs.) With just-in-time inputs, given the relative precisions of signals captured by r_A and r_θ, the magnitude of aggregate fluctuations in period t in response to horizon-T signals in $\mathcal{M}_t$ is determined by $\widetilde{D_T}$, defined as*

$$(\widetilde{D_T})^2 = (\beta^T)^2\left(r_A\sigma_A^2(\widetilde{\overline{d^T}})^2 + n^{-(\delta-1)}r_\theta\sigma_\theta^2\widetilde{V^{in,T}}\right).$$

then aggregate fluctuations in period t in response to horizon-T signals observed in t satisfy

$$\sigma(\widehat{c}_t) \geq \frac{1}{\widetilde{d^{out,\,all}_{\max}}}\left(1 - \frac{\gamma}{1-\alpha_{0,\max}}\right)\widetilde{D_T}, \qquad \sigma(\widehat{h}_t) \geq \frac{1}{\widetilde{\overline{d^{all}}}}\frac{1-\alpha_{0,\max}}{1-\alpha_{0,\min}}\widetilde{D_T}, \qquad \sigma(\widehat{q}_t) \geq \frac{\gamma}{1-\alpha_{0,\min}}\widetilde{D_T},$$

where $\widetilde{d^{out,\,all}_{\max}} = \max_{1\leq i\leq n}\widetilde{d^{out,\,all}_i}$, $\widetilde{\overline{d^{all}}} = \frac{1}{n}\sum_i \widetilde{d^{out,all_i}}$, $\alpha_{0,\max} = \max_{1\leq i\leq n}\alpha_{0,i}$, *and* $\alpha_{0,\min} = \min_{1\leq i\leq n}\alpha_{0,i}$.

Obtaining the lower bounds follows the same procedure as in A.2 but requires minor changes. For non-negatively correlated fundamentals, we have

$$\left.\frac{\mathrm{d}\boldsymbol{s}_t}{\mathrm{d}A_{t+T|t}}\right|_{SS} \geq \frac{1}{A}\boldsymbol{L}_0'(\beta\boldsymbol{\Omega}_1'\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{L}_0')^T\boldsymbol{\theta} \quad \text{and} \quad \left.\frac{\mathrm{d}\boldsymbol{s}_t}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS} \geq \boldsymbol{L}_0'(\beta\boldsymbol{\Omega}_1'\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{L}_0')^T.$$

Adjusting for uses of jit inputs, the aggregate final use can be written as

$$\begin{aligned}
\ln C_t =& \boldsymbol{\theta}'\left(\boldsymbol{L}_0[\gamma + \operatorname{diag}(\boldsymbol{\alpha}_0) - \operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0] - \boldsymbol{I}\right)\ln\boldsymbol{s}_t + \Psi_C^{En}(t-1) + \Psi_C^{Ex}(t) \\
=& \boldsymbol{\theta}'\left(\gamma\boldsymbol{L}_0 + \boldsymbol{L}_0\operatorname{diag}(\boldsymbol{\alpha}_0) - \boldsymbol{L}_0\right)\ln\boldsymbol{s}_t + \Psi_C^{En}(t-1) + \Psi_C^{Ex}(t)
\end{aligned}$$

where, as before, $\Psi_C^{En}(t-1)$ and $\Psi_C^{Ex}(t)$ contain endogenous and exogenous variables irrelevant for the result. The second equality uses the fact that $\boldsymbol{L}_0 = \boldsymbol{I} + \boldsymbol{L}_0\operatorname{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0$. Furthermore,

observe that $\gamma \boldsymbol{L}_0 + \boldsymbol{L}_0 \operatorname{diag}(\boldsymbol{\alpha}_0) - \boldsymbol{L}_0 \leq 0$. It follows that

$$\begin{aligned}
\left| \frac{\mathrm{d} \ln C_t}{\mathrm{d} A_{t+T|t}} \Big|_{SS} \right| \geq & \mathbf{1}' \left((1-\gamma) \boldsymbol{L}_0 - \boldsymbol{L}_0 \operatorname{diag}(\boldsymbol{\alpha}_0) \right) \operatorname{diag}(\boldsymbol{L}_0' \widetilde{\boldsymbol{d}^{\text{out, all}}})^{-1} \frac{1}{A} \boldsymbol{L}_0' (\beta \boldsymbol{\Omega}_1' \operatorname{diag}(\boldsymbol{\alpha}_1) \boldsymbol{L}_0')^T \frac{1}{n} \mathbf{1} \\
\geq & \frac{\beta^T}{n A d_{\max}^{\text{out, all}}} \left((1-\gamma) (\boldsymbol{L}_0' \mathbf{1})' - (\boldsymbol{L}_0' \mathbf{1})' \operatorname{diag}(\boldsymbol{\alpha}_0) \right) \operatorname{diag}(\boldsymbol{L}_0' \mathbf{1})^{-1} \boldsymbol{L}_0' \boldsymbol{d}^{\text{out},T} \\
= & \frac{\beta^T}{n A d_{\max}^{\text{out, all}}} \left((1-\gamma) \mathbf{1}' - \boldsymbol{\alpha}_0' \right) \boldsymbol{L}_0' \boldsymbol{d}^{\text{out},T} \\
\geq & \frac{\beta^T}{n A d_{\max}^{\text{out, all}}} \left(\mathbf{1}' - \gamma (\boldsymbol{L}_0 \mathbf{1})' \right) \boldsymbol{d}^{\text{out},T} \geq \frac{\beta^T}{A d_{\max}^{\text{out, all}}} \left(1 - \frac{\gamma}{1 - \alpha_{0,\max}} \right) \widetilde{\widetilde{d^T}}.
\end{aligned}$$

The last two inequalities exploit the following facts: $\boldsymbol{L}_0 \boldsymbol{\alpha}_0 = \boldsymbol{L}_0 \mathbf{1} - \mathbf{1}$, and $\boldsymbol{L}_0 \mathbf{1} = \mathbf{1} + \boldsymbol{\alpha}_0 + \boldsymbol{\Omega}_0 \boldsymbol{\alpha}_0 + \ldots \leq (1 + \alpha_{0,\max} + \alpha_{0,\max}^2 + \ldots) \mathbf{1} = \frac{1}{1-\alpha_{0,\max}} \mathbf{1}$. In the same way, we have

$$\begin{aligned}
\left| \frac{\mathrm{d} \ln C_t}{\mathrm{d} \boldsymbol{\theta}_{t+T|t}} \Big|_{SS} \right| \geq & \mathbf{1}' \left((1-\gamma) \boldsymbol{L}_0 - \boldsymbol{L}_0 \operatorname{diag}(\boldsymbol{\alpha}_0) \right) \operatorname{diag}(\boldsymbol{L}_0' \widetilde{\boldsymbol{d}^{\text{out, all}}})^{-1} \boldsymbol{L}_0' (\beta \boldsymbol{\Omega}_1' \operatorname{diag}(\boldsymbol{\alpha}_1) \boldsymbol{L}_0')^T \\
\geq & \frac{\beta^T}{d_{\max}^{\text{out, all}}} \left(1 - \frac{\gamma}{1 - \alpha_{0,\max}} \right) (\widetilde{\boldsymbol{d}^{\text{in},T}})'.
\end{aligned}$$

The lower bound for volatility in $\widehat{c}_t$ follows.

As before, $\widehat{h}_t$ is much simpler. Observe that $\frac{1}{1-\alpha_{0,\max}} \mathbf{1} \geq \boldsymbol{L}_0 \mathbf{1} = \mathbf{1} + \boldsymbol{\alpha}_0 + \boldsymbol{\Omega}_0 \boldsymbol{\alpha}_0 + \ldots \geq \frac{1}{1-\alpha_{0,\min}} \mathbf{1}$. Then,

$$\begin{aligned}
\frac{\mathrm{d} \ln H_t}{\mathrm{d} A_{t+T|t}} \Big|_{SS} \geq & \frac{1}{\mathbf{1}' \boldsymbol{s}} \frac{\mathrm{d} \boldsymbol{s}_t}{\mathrm{d} A_{t+T|t}} = \frac{1}{A \frac{1}{n} \mathbf{1}' \boldsymbol{L}_0' \widetilde{\boldsymbol{d}^{\text{out, all}}}} \mathbf{1}' \boldsymbol{L}_0' (\beta \boldsymbol{\Omega}_1' \operatorname{diag}(\boldsymbol{\alpha}_1) \boldsymbol{L}_0')^T \boldsymbol{\theta} \\
\geq & \frac{\beta^T}{A n \frac{1}{1-\alpha_{0,\max}} \widetilde{\widetilde{d^{\text{all}}}}} \frac{1}{1 - \alpha_{0,\min}} [(\boldsymbol{L}_0 \operatorname{diag}(\boldsymbol{\alpha}_1) \boldsymbol{\Omega}_1)^T \mathbf{1}]' \mathbf{1} = \frac{\beta^T}{A \widetilde{\widetilde{d^{\text{all}}}}} \frac{1 - \alpha_{0,\max}}{1 - \alpha_{0,\min}} \widetilde{\widetilde{d^T}}, \\
\frac{\mathrm{d} \ln H_t}{\mathrm{d} \boldsymbol{\theta}_{t+T|t}} \Big|_{SS} \geq & \frac{1}{\frac{1}{n} \mathbf{1}' \boldsymbol{L}_0' \widetilde{\boldsymbol{d}^{\text{out, all}}}} \mathbf{1}' \boldsymbol{L}_0' (\beta \boldsymbol{\Omega}_1' \operatorname{diag}(\boldsymbol{\alpha}_1) \boldsymbol{L}_0')^T = \frac{\beta^T}{\widetilde{\widetilde{d^{\text{all}}}}} \frac{1 - \alpha_{0,\max}}{1 - \alpha_{0,\min}} (\widetilde{\boldsymbol{d}^{\text{in},T}})'.
\end{aligned}$$

The lower bound for volatility in $\widehat{h}_t$ follows.

For $\widehat{q}_t$, we start from the price adjustment term. As before, I ignore the details of the irrelevant variables and use $\Psi_P^{En}(t-1, t-2)$ and $\Psi_P^{Ex}(t, t-1)$ to summarize them. Then,

$$\begin{aligned}
& (\boldsymbol{s}_t \oslash \boldsymbol{s}_{t-1}) \oslash (\boldsymbol{y}_t \oslash \boldsymbol{y}_{t-1}) = \exp(\Delta \ln \boldsymbol{s}_t - \Delta \ln \boldsymbol{y}_t) \\
= & \exp \left([\boldsymbol{I} - \gamma \boldsymbol{L}_0 - \boldsymbol{L}_0 \operatorname{diag}(\boldsymbol{\alpha}_0) + \boldsymbol{L}_0 \operatorname{diag}(\boldsymbol{\alpha}_0) \boldsymbol{\Omega}_0] \Delta \ln \boldsymbol{s}_t + \Psi_P^{En}(t-1, t-2) + \Psi_P^{Ex}(t, t-1) \right) \\
= & \exp \left([(1-\gamma) \boldsymbol{L}_0 - \boldsymbol{L}_0 \operatorname{diag}(\boldsymbol{\alpha}_0)] \Delta \ln \boldsymbol{s}_t + \Psi_P^{En}(t-1, t-2) + \Psi_P^{Ex}(t, t-1) \right).
\end{aligned}$$

Consequently, when $t-1$ is at SS,

$$\begin{aligned}
&\left.\frac{\mathrm{d}\ln Q_t}{\mathrm{d}\boldsymbol{s}_t}\right|_{SS}\\
=&\left.\frac{\mathrm{d}\ln C_t}{\mathrm{d}\boldsymbol{s}_t}\right|_{SS}+1\times\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{s}_t}\big((\mathbf{1}-\boldsymbol{\alpha}_0)'\boldsymbol{s}_t-[(\boldsymbol{I}-\boldsymbol{\Omega}_0'\operatorname{diag}(\boldsymbol{\alpha}_0))\boldsymbol{s}-\boldsymbol{\theta}]'[(\boldsymbol{s}_t\oslash\boldsymbol{s})\oslash(\boldsymbol{y}_t\oslash\boldsymbol{y})\big)\Big|_{SS}\\
=&\boldsymbol{\theta}'\left(\gamma\boldsymbol{L}_0+\boldsymbol{L}_0\operatorname{diag}(\boldsymbol{\alpha}_0)-\boldsymbol{L}_0\right)\operatorname{diag}(\boldsymbol{s})^{-1}+(\mathbf{1}-\boldsymbol{\alpha}_0)'\\
&-[(\boldsymbol{I}-\boldsymbol{\Omega}_0'\operatorname{diag}(\boldsymbol{\alpha}_0))\boldsymbol{s}-\boldsymbol{\theta}]'\boldsymbol{I}[(1-\gamma)\boldsymbol{L}_0-\boldsymbol{L}_0\operatorname{diag}(\boldsymbol{\alpha}_0)]\operatorname{diag}(\boldsymbol{s})^{-1}\\
=&\gamma\mathbf{1}'.
\end{aligned}$$

Finally, using the fact that $\boldsymbol{L}_0\mathbf{1}\geq\frac{1}{1-\alpha_{0,\min}}\mathbf{1}$, and we have

$$\begin{aligned}
\left.\frac{\mathrm{d}\ln Q_t}{\mathrm{d}A_{t+T|t}}\right|_{SS}\geq&\frac{\beta^T\gamma}{nA}\mathbf{1}'\boldsymbol{L}_0'(\boldsymbol{\Omega}_1'\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{L}_0')^T\mathbf{1}\geq\frac{\beta^T\gamma}{A(1-\alpha_{0,\min})}\widetilde{\widetilde{d^T}},\\
\left.\frac{\mathrm{d}\ln Q_t}{\mathrm{d}\boldsymbol{\theta}_{t+T|t}}\right|_{SS}\geq&\beta^T\gamma\mathbf{1}'\boldsymbol{L}_0'(\boldsymbol{\Omega}_1'\operatorname{diag}(\boldsymbol{\alpha}_1)\boldsymbol{L}_0')^T\geq\frac{\beta^T\gamma}{1-\alpha_{0,\min}}(\widetilde{\boldsymbol{d}^{\mathrm{in},T}})'.
\end{aligned}$$

The lower bound for volatility in $\widehat{q}_t$ follows.

B.1.4 General Case

More generally, input goods may require b periods of time to build, such that $b\in B=\{0,1,\ldots,\overline{b}\}$ for some finite $\overline{b}\in\mathbb{N}$. Firm i's production function is assumed to be

$$y_{i,t}=z_{i,t}h_{i,t}^{\gamma_i}\prod_{b\in B}X_{i,b,t-b}^{\alpha_{i,b}},\quad \gamma_i+\sum_b\alpha_{i,b}\leq 1,$$

where $X_{i,b,t-b}$ is the input compound that requires b-periods to build and is purchased b periods in advance, such that

$$X_{i,b,t-b}=\prod_j x_{ij,b,t-b}^{\omega_{ij,b}},\quad \text{with }\omega_{ij,b}\geq 0,\quad \sum_j\omega_{ij,b}=1,\quad\forall\, b\in B.$$

Denote $\boldsymbol{\Omega}_b=[\omega_{ij,b}]$ and $\boldsymbol{\alpha}_b=(\alpha_{i,b})$, for each $b\in B$. For every non-zero element $\omega_{ij,b}$ in $\boldsymbol{\Omega}_b$, firm i purchases input from firm j, and it takes b periods for this input to transform into firm i's output. The conventional input-output matrix is therefore split into $\overline{b}+1$ parts based on time to build, $\operatorname{diag}(\boldsymbol{\alpha}_b)\boldsymbol{\Omega}_b$, $b\in B$. Each of these sub-matrices describes a weighted directed graph such that every edge requires b periods to complete.

Each firm's flow payoff has a similar form such that

$$\pi_{i,t}=p_{i,t}y_{i,t}-w_th_{i,t}-\sum_b\sum_j p_{j,t}x_{ij,b,t},$$

and the firm maximizes discounted expected profit stream $\mathbb{E}_t\left[\sum_{t'\geq t}\lambda_{t,t'}\pi_{i,t'}\right]$. The Euler equations

evaluated at the equilibrium imply

$$\frac{p_{j,t}x_{ij,b,t}}{P_tC_t} = \frac{1}{A_t}\beta^b\alpha_{i,b}\omega_{ij,b}\,\mathbb{E}_t[A_{t+b}s_{i,t+b}], \quad \forall\, i,j,b.$$

The market clearing condition for each good becomes

$$y_{i,t} = c_{i,t} + \sum_{b\in B}\sum_j x_{ji,b,t}$$

which implies $s_{i,t} = \theta_{i,t} + \sum_{b\in B}\beta^b\sum_j \alpha_{j,b}\omega_{ji,b}\frac{1}{A_t}\,\mathbb{E}_t[A_{t+b}s_{j,t+b}]$. Writing it in vectors and matrices yields

$$\boldsymbol{s}_t = \boldsymbol{\theta}_t + \frac{1}{A_t}\sum_{b\in B}\beta^b\boldsymbol{\Omega}_b'\,\mathrm{diag}(\boldsymbol{\alpha}_b)\,\mathbb{E}_t[A_{t+b}\boldsymbol{s}_{t+b}] = \boldsymbol{L}_0'\left(\boldsymbol{\theta}_t + \frac{1}{A_t}\sum_{b\in B^+}\beta^b\boldsymbol{\Omega}_b'\,\mathrm{diag}(\boldsymbol{\alpha}_b)\,\mathbb{E}_t[A_{t+b}\boldsymbol{s}_{t+b}]\right),$$

where $\boldsymbol{L}_0 = [\boldsymbol{I} - \mathrm{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0]^{-1}$ is the Leontief inverse of $\mathrm{diag}(\boldsymbol{\alpha}_0)\boldsymbol{\Omega}_0$ as defined before, and $B^+ \equiv B\setminus\{0\}$. At the deterministic steady state, equilibrium Domar weights become $\boldsymbol{s} = \left[\boldsymbol{I} - \sum_{b\in B}\beta^b\boldsymbol{\Omega}_b'\,\mathrm{diag}(\boldsymbol{\alpha}_b)\right]^{-1}\boldsymbol{\theta}$. In general, the equilibrium Domar weights still take the form of time and network weighted final expenditure shares, such that

$$\begin{aligned}
\boldsymbol{s}_t =& \boldsymbol{L}_0'\boldsymbol{\theta}_t + \boldsymbol{L}_0'\sum_{b\in B^+}\beta^b\boldsymbol{\Omega}_b'\,\mathrm{diag}(\boldsymbol{\alpha}_b)\frac{1}{A_t}\times \\
&\mathbb{E}_t\left[A_{t+b}\left(\boldsymbol{L}_0'\boldsymbol{\theta}_{t+b} + \boldsymbol{L}_0'\sum_{bb\in B^+}\beta^{bb}\boldsymbol{\Omega}_{bb}'\,\mathrm{diag}(\boldsymbol{\alpha}_{bb})\frac{1}{A_{t+b}}\,\mathbb{E}_{t+b}[A_{t+b+bb}\boldsymbol{s}_{t+b+bb}]\right)\right] \\
=& \boldsymbol{L}_0'\boldsymbol{\theta}_t + \boldsymbol{L}_0'\sum_{b\in B^+}\beta^b\boldsymbol{\Omega}_b'\,\mathrm{diag}(\boldsymbol{\alpha}_b)\frac{1}{A_t}\,\mathbb{E}_t[A_{t+b}\boldsymbol{L}_0'\boldsymbol{\theta}_{t+b}] \\
&+ \boldsymbol{L}_0'\sum_{b\in B^+}\beta^b\boldsymbol{\Omega}_b'\,\mathrm{diag}(\boldsymbol{\alpha}_b)\boldsymbol{L}_0'\sum_{bb\in B^+}\beta^{bb}\boldsymbol{\Omega}_{bb}'\,\mathrm{diag}(\boldsymbol{\alpha}_{bb})\frac{1}{A_t}\,\mathbb{E}_t[A_{t+b+bb}\boldsymbol{L}_0'\boldsymbol{\theta}_{t+b+bb}] + \ldots \\
=& \boldsymbol{L}_0'\boldsymbol{\theta}_t + \frac{1}{A_t}\boldsymbol{L}_0'\sum_{\tau=1}^{\infty}\beta^\tau\boldsymbol{\Gamma}_\tau'A_{t+\tau|t}\boldsymbol{\theta}_{t+\tau|t},
\end{aligned}$$

where

$$\boldsymbol{\Gamma}_\tau \equiv \sum_{\substack{k\le\tau,\ \{b^i\}_{i=1}^k \text{ s.t.} \\ b^i\in B^+ \text{ and } \sum_{i=1}^k b^i=\tau}} (\boldsymbol{L}_0\,\mathrm{diag}(\boldsymbol{\alpha}_{b^1})\boldsymbol{\Omega}_{b^1}) \times \ldots \times (\boldsymbol{L}_0\,\mathrm{diag}(\boldsymbol{\alpha}_{b^k})\boldsymbol{\Omega}_{b^k}).$$

Here, $\boldsymbol{\Gamma}_\tau$ enumerates every weighted walk from one firm to another that takes a total of τ periods to complete. Therefore, it contains all the possible ways for signals about final demand in τ periods to have immediate impacts on the economy. In the simple 0-or-1 case and the homogeneous time to build case, there is a sense of monotonicity of the impact of signals and shocks in τ: signals about the future tend to have smaller impacts on the current outcome if the future is more distant, and impacts of realized shocks now tend to diminish in the following periods. However, a similar sense of monotonicity may not exist in the more general case. This is because, for example, a path

that takes 2 periods to complete includes edges that take at most 2 periods to complete ($\alpha_b \omega_{ij,b} > 0,\ b = 0, 1, 2$), whereas a 5-period path can include all edges that take up to 5 periods to complete ($\alpha_b \omega_{ij,b} > 0,\ b = 0, 1, \ldots, 5$). Despite heavier discounting ($\beta^5 < \beta^2$), the combined weight of the latter may dominate that of the former. In that case, the economy responds more to signals about more distant future because a larger set of industries have enough time to react; and disturbances in a more distant past affect the current economy more because some spillover effects take time to realize.

Because of the possibility of multiple periods to build, the law of motion of industrial output growths becomes

$$\begin{aligned}\ln \boldsymbol{y}_t =& \boldsymbol{L}_0 \Bigg[\sum_{b \geq 1} \operatorname{diag}(\boldsymbol{\alpha}_b) \boldsymbol{\Omega}_b \ln \boldsymbol{y}_{t-b} + \ln \boldsymbol{z}_t + \boldsymbol{\gamma} \ln A_t \\ &+ [\operatorname{diag}(\boldsymbol{\gamma}) + \operatorname{diag}(\boldsymbol{\alpha}_0)(\boldsymbol{I} - \boldsymbol{\Omega}_0)] \ln \boldsymbol{s}_t + \sum_{b \geq 1} \operatorname{diag}(\boldsymbol{\alpha}_b) \big(\ln(\mathbb{E}_{t-b}[A_t \boldsymbol{s}_t] / A_{t-b}) - \boldsymbol{\Omega}_b \ln \boldsymbol{s}_{t-b} \big) + \boldsymbol{\psi} \Bigg].\end{aligned}$$

The intertemporal downstream transmission of realized impacts of shocks is not necessarily monotone in time either.

I omit the more general version of Theorem 4. Details are available upon request.

B.2 Import-export Adjustment

For parameterization and quantitative explorations, I modify the baseline model to accommodate import-export in the input-output tables. The simplest way to do so is in a reduced form by introducing another parameter vector $\boldsymbol{\xi} = (\xi_i)$, where $\xi_i > 0$ determines the discrepancy between industry i's gross output and total expenditure on i's product within the country; $\xi_i < 1$ means net import of product i and $\xi_i > 1$ net export. If industry i's total output is $y_{i,t}$, then the market clearing condition for good i becomes

$$y_{i,t} \xi_i^{-1} = c_{i,t} + \sum_{j=1}^{n} x_{ji,t}.$$

An implicit assumption here is that the import or export of a good affects both its domestic final use and its domestic input use, and by the same proportion.

Households' income changes accordingly and becomes $w_t h_t + \Pi_t - NX_t$ where $NX_t = \sum_{i=1}^{n} p_{i,t} y_{i,t} (1 - \xi_{i,t}^{-1})$ is the aggregate net export. In equilibrium, we still have $p_{i,t} c_{i,t} = \theta_{i,t} P_t C_t$ but now the budget constraint implies $P_t C_t = w_t h_t + \Pi_t - NX_t$. This leads to the main change of the model, and equation (1) becomes

$$\boldsymbol{s}_t = \operatorname{diag}(\boldsymbol{\xi}) \left[\boldsymbol{\theta}_t + \frac{1}{A_t} \beta \boldsymbol{\Omega}' \operatorname{diag}(\boldsymbol{\alpha}) \, \mathbb{E}_t[A_{t+1} \boldsymbol{s}_{t+1}] \right].$$

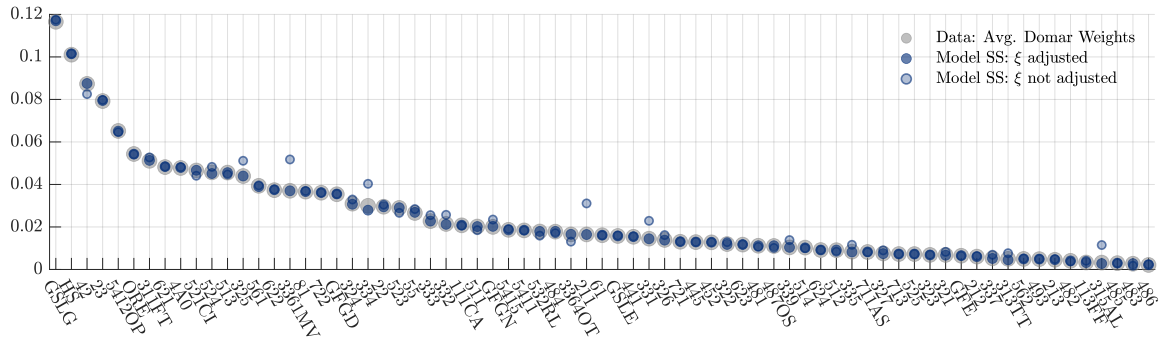


Figure B.1: Model-implied steady-state Domar weights $\boldsymbol{s}$ versus data, with and without import-export adjustment parameter $\boldsymbol{\xi}$.

Equation (2) in Theorem 1 becomes

$$\boldsymbol{s}_t = \operatorname{diag}(\boldsymbol{\xi})\boldsymbol{\theta}_t + \frac{1}{A_t}\operatorname{diag}(\boldsymbol{\xi})\sum_{\tau\geq 1}(\beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha})\operatorname{diag}(\boldsymbol{\xi}))^{\tau}A_{t+\tau|t}\boldsymbol{\theta}_{t+\tau|t}.$$

At the deterministic steady states, Domar weights satisfy

$$\boldsymbol{s} = (\operatorname{diag}(\boldsymbol{\xi})^{-1} - \beta\boldsymbol{\Omega}'\operatorname{diag}(\boldsymbol{\alpha}))^{-1}\boldsymbol{\theta}.$$

The relationship between consumption, input, and output remains the same, i.e., $c_{i,t} = y_{i,t}\frac{\theta_{i,t}}{s_{i,t}}$ and $x_{ji,t} = y_{i,t}\frac{\beta\alpha_j\omega_{ji}\mathbb{E}_t[A_{t+1}s_{j,t+1}]}{A_t s_{i,t}}$. And so the aggregate final use still satisfies $\ln C_t = \boldsymbol{\theta}'(\ln \boldsymbol{y}_t + \ln \boldsymbol{\theta}_t - \ln \boldsymbol{s}_t)$, whereas the real GDP defined in Section 4 now becomes

$$\begin{aligned} Q_t =& C_t\left(\sum_i s_{i,t} - \sum_i \frac{s_{i,t}/s_{i,t-1}}{y_{i,t}/y_{i,t-1}}\left[s_{i,t-1}\xi_i^{-1} - \theta_{i,t-1}\right]\right) \\ =& C_t\left(\mathbf{1}'\boldsymbol{s}_t - [\boldsymbol{s}_{t-1}\oslash\boldsymbol{\xi} - \boldsymbol{\theta}_{t-1}]'[(\boldsymbol{s}_t\oslash\boldsymbol{s}_{t-1})\oslash(\boldsymbol{y}_t\oslash\boldsymbol{y}_{t-1})]\right). \end{aligned}$$

At the deterministic steady state, $Q = C(1 + (\mathbf{1} - \boldsymbol{\xi}^{-1})'\boldsymbol{s})$, where $\boldsymbol{\xi}^{-1} = (\xi_i^{-1})$. The rest of the model remains unchanged. Figure B.1 compares the fit of the model at its steady state with or without such adjustment. Calibration of $\boldsymbol{\xi}$ is in C.

Extending the model to allow for time-varying $\boldsymbol{\xi}_t$ or import-export shocks as another source of fluctuation is straightforward. Introducing noisy signals about such shocks in the future is also feasible, without altering the structure or the tractability of the model. I omit the details, which are available upon request.

B.3 Government Expenditures

The baseline model does not consider governments explicitly, for two reasons. First, the baseline model is intended to be free of all frictions, market imperfections, and distortions. And second, according to NIPAs and the BEA, the general government industry is now acknowledged as a pro-

ducer in the input-output table (Karen J. Horowitz, 2009). Therefore, when the model is mapped to data in C, governments are treated in a similar way to private industries, which use intermediate inputs to provide services.

Conventionally, government expenditures are considered as part of the aggregate final use only, per the GDP accounting rule. I will show that the baseline model setup is without loss of generality and time-varying final-use shares $\{\boldsymbol{\theta}_t\}$ may reflect changes in industry-specific government (final-use) expenditures as well.

Suppose that a government exists in the baseline economy. Its industry-specific expenditures in period t is government by an exogenous, random plan $\boldsymbol{g}_t = (g_{i,t}) \in \mathbb{R}^n_+$ with finite first and second moments. The government spends $g_{j,t} \sum_i p_{i,t} c_{i,t}$ to buy good j. These purchased goods are then destroyed. To finance the expenditure, the government levies a non-distortionary lump sum tax T_t on the household. The household's budget constraint becomes $\sum_i p_{i,t} c_{i,t} \leq w_t h_t + \Pi_t - T_t$. A balanced government budget requires $\sum_j g_{j,t} \sum_i p_{i,t} c_{i,t} = T_t$.

In addition to Assumption 1, I further assume similar independence of $\boldsymbol{g}_t$ such that $\forall t$, and $t', t'' > t$, $\boldsymbol{g}_{t'} \perp\!\!\!\perp A_{t''}$, and $\boldsymbol{g}_{t'} \perp\!\!\!\perp A_{t''} \mid (\mathcal{M}_\tau)_{\tau \leq t}$. Hence, in this case, $\mathcal{M}_t$ may contain signals about future government expenditures. The market clearing condition then implies

$$\begin{aligned}\boldsymbol{s}_t =& \boldsymbol{\theta}_t + \boldsymbol{g}_t + \frac{1}{A_t} \beta \boldsymbol{\Omega}' \operatorname{diag}(\boldsymbol{\alpha}) \mathbb{E}_t[A_{t+1} \boldsymbol{s}_{t+1}] \\ =& \boldsymbol{\theta}_t + \boldsymbol{g}_t + \frac{1}{A_t} \sum_{\tau \geq 1} (\beta \boldsymbol{\Omega}' \operatorname{diag}(\boldsymbol{\alpha}))^\tau A_{t+\tau|t} (\boldsymbol{\theta}_{t+\tau|t} + \boldsymbol{g}_{t+\tau|t}).\end{aligned}$$

This suggests that $\boldsymbol{\theta}_t$ and $\boldsymbol{g}_t$, together with their signals, affect Domar weights in the same way. The lump sum tax does not distort labor supply and so the law of motion industrial output in equation (3) does not change. The aggregate final use consists of aggregate consumption C_t and aggregate government expenditure $C_t(\mathbf{1}'\boldsymbol{g}_t)$. After adjusting for input prices, the aggregate real value added can be written as $Q_t = C_t\big(\mathbf{1}'\boldsymbol{s}_t - (\boldsymbol{s}_{t-1} - \boldsymbol{\theta}_{t-1} - \boldsymbol{g}_{t-1})'[(\boldsymbol{s}_t \oslash \boldsymbol{s}_{t-1}) \oslash (\boldsymbol{y}_t \oslash \boldsymbol{y}_{t-1})]\big)$. At the deterministic steady state, $Q = C(1 + \mathbf{1}'\boldsymbol{g})$.

The symmetric roles of $\boldsymbol{\theta}_{t+\tau|t}$ and $\boldsymbol{g}_{t+\tau|t}$ imply that the volatility lower bounds in Theorem 2 apply to the modified model with minimal changes. In fact, if the government expenditure plan is symmetric across industries, with uniform unconditional expectations $\boldsymbol{g} = \frac{G}{n}\mathbf{1}$ for some $G \geq 0$, then $\boldsymbol{s} = \frac{1+G}{n} \boldsymbol{d}^{\text{out, all}}$. It follows that $\frac{\mathrm{d} \ln Q_t}{\mathrm{d} \boldsymbol{g}_{t+T|t}}\big|_{SS} \geq \frac{\beta^T \gamma}{1+G} (\boldsymbol{d}^{\text{in},T})'$ when $\{\boldsymbol{g}_t\}$ has non-negative serial correlation. The volatility in aggregate value added caused by information about future government expenditures has a lower bound proportional to $V^{\text{in},T}$. The lower bound is achieved when $\{\boldsymbol{g}_t\}$ is i.i.d. over time but has a negative cross-sectional correlation coefficient $-\frac{1}{n-1}$. I omit the details of proofs.

In summary, the modeling choice of $\boldsymbol{\theta}_t$ as a source of fluctuation is without loss of generality. It includes shocks to industry-specific government final-use expenditures and relevant signals as a special case. The baseline model also accommodates the role of a government as a producer, featuring a distinct set of input-output linkages with other industries.

C Data and Parameterization

C.1 Data Description

The primary source of data is the BEA industry data for the U.S. economy. Input-output matrices are based on the annual I-O make and use tables (before redefinitions) in *producer* price. NAICS coded tables at the sector (number of industries $n = 15$) and summary levels ($n = 71$) are available annually from 1997 to 2018; tables at the detail level ($n = 405$) are available for 2007 and 2012. Quarterly data on gross output and intermediate input expenditure by industry are from 2005Q1 to 2019Q4 at *summary* level with 71 industries. Quarterly data on aggregate GDP and its relevant components (labor compensation, net export) are from BEA's national account (NIPA). Quarterly data on total hours worked of all sectors are from the U.S. Bureau of Labor Statistics's Labor Productivity and Costs.

The I-O tables in the conventional make-use framework are available both in producer price and in purchase price. Between these two choices, the make and use tables in *producers'* values are preferable to those in purchasers' values. The reasons are as follows. Trade and transportation margins are a main contributor to the difference between a product's value paid by its purchasers and the same product's value received by its producer. These margins are part of the purchasers' expenditures, and they eventually become payments to corresponding industries along the physical supply chains (e.g., transportation, warehousing, wholesale and retail trades, etc.) instead of the producer. Therefore, although purchaser values better reflect the actual costs of using each commodity, they do not sufficiently acknowledge the importance of the industries that charge these margins. In contrast, the make and use tables in producer values attribute these margins to those industries, and consequently parameters based on these tables put more weights on the physical supply chains (the trade and transportation industries) in the production network. For the same reason, to calibrate this model, the conventional make-use framework is preferable to the new BEA supply-use framework adopted since 2018. The use tables in the new framework are in purchasers' values.

C.2 Variable Construction

The BEA provides data on quarterly gross sales $Sales_{i,t}$ and input expenditure $InputExp_{i,t}$ by industry in \$. To obtain the data counterpart of $\boldsymbol{s}_t$ and $\boldsymbol{u}_t$, I divide them by concurrent quarterly final use measure FU_t, i.e. the data counterpart of $P_t C_t$. Taking into consideration import-export adjustment, I define nominal final use as nominal GDP with net export removed for each year-quarter t,

$$FU_t = GDP_t - NX_t,$$

where GDP_t is nominal GDP in \$ and NX_t nominal net export in \$. In other words, using the industry IO codes in BEA tables, FU_t includes F010+F020+F030+F100, and NX_t is F040+F050.

Therefore,

$$s_{i,t}^{data} = \frac{Sales_{i,t}}{FU_t}, \text{ and } u_{i,t}^{data} = \frac{InputExp_{i,t}}{FU_t}.$$

Note that, directly using nominal GDP_t instead of FU_t as the data counterpart of P_tC_t does *not* lead to significant changes in volatility in $\left\{s_{i,t}^{data}, u_{i,t}^{data}\right\}$.

There is no one-to-one mapping between an industry and a commodity, and I need to utilize the I-O Make tables (Industry-by-Commodity, $Make_t(i,j)$ = industry i's production of commodity j in \$ at t) to convert information in Use tables (Commodity-by-Industry, $Use_t(i,j)$ = industry(or final user) j's input (or final) use of commodity i in \$ at t) into industry-by-industry variables. Assume each industry contributes to each commodity proportionally, according to the Make tables. Define matrix $make_t$ for each year t such that

$$make_t(i,j) = \frac{Make_t(i,j)}{\sum_k Make_t(k,j)}.$$

Then for each year t, an industry's input expenditure on another industry's product is

$$\text{industry } i\text{'s use of industry } j\text{'s output}_t = make_t(j,:) \times Use_t(:,i);$$

domestic final users' expenditure on an industry is

$$\text{final-use expenditure on industry } i\text{'s output}_t = make_t(i,:) \times Use_t(:, \text{F010+F020+F030+F100});$$

import-export adjustments by industry is

$$\text{net export of industry } i\text{'s output}_t = make_t(i,:) \times Use_t(:, \text{F040+F050}).$$

C.3 Parameters with Direct Targets

For tractability, this paper does not model explicitly capital and its dynamics. This is perhaps less restrictive than one might think. The model assumes Cobb-Douglas production functions with non-increasing returns to scale. Under this assumption, the model is equivalent to one where capital (or any primary input factor other than labor) is industry-specific and in fixed supply. Capital enters the production of good i with an exponent no higher than $1 - \alpha_i - \gamma_i, \forall\, i$. Households own the capital and receive both the profit and the capital rents from firms. There is no change to the households' budget constraints in equilibrium or other equilibrium conditions written in expenditures or growth rates. The reason is that, just as productivity levels do not affect expenditures, capital levels do not alter expenditure on labor or inputs. Hence, without loss of generality, the fixed supply of each industry-specific capital can be normalized to one.

In the model's current specification, the data counterpart of profit $\pi_{i,t}$ of an industry is the gross output less input expenditure and labor compensation. This profit measure then includes all costs other than labor compensation as well as the actual profit. It follows that the ratios of labor compensation over gross industry output are a direct match to $\boldsymbol{\gamma}$ in the model. Similarly, the ratios of total input expenditures over gross industry output are a match to $\boldsymbol{\alpha}$. In the same spirit, the vector of final-use shares $\boldsymbol{\theta}$ corresponds to the distribution of total final-use expenditures on every

industry's output, including personal consumption expenditures and private fixed investments, among other things.

The advantage of such a match between model and data is a clear division of users into input users (industries) and final users (households in the model). It follows that the output of each industry flows to two clear and mutually exclusive destinations: one is the households and the flow stops (part of $\boldsymbol{\theta}$); the other is all the industries and the flow continues but as a different kind of output (part of the I-O matrix $\text{diag}(\boldsymbol{\alpha})\boldsymbol{\Omega}$). It is then much easier to use the I-O tables and other data from the BEA industry account. The match turns out to be very accurate despite the lack of explicit capital accumulation. Figure B.1 compares the model-implied steady-state Domar weights using such calibrated parameters and the average Domar weights directly calculated from data. It is reassuring that these two vectors are indeed very close, which is not surprising through the lens of the model. The model indicates that the two uses (final and input) of industrial output affect Domar weights in very different ways. Therefore, it is more important to distinguish between a product's final use and its input use when matching the data. The detailed allocation — consumption or investment — within the final use is less crucial.

Note that such treatment of investments weakens the model mechanism in two ways. First, by assumption, final uses are not subject to the time-to-build, whereas, in reality, capital accumulation (especially structures) likely takes a longer time than any intermediate inputs. For example, a new building may take months or years to finish, but legal services or software designs take days or weeks. Therefore, a conventional channel — (aggregate) capital accumulation dynamics — for signals about the future to have current impacts is missing. Second, capital flows among industries are also missing. These transactions strengthen industries interconnections due to input-output linkages, which amplify and prolong the signals' impact. Further exploration is in C.5 using the BEA's 1997 capital flow table.

In summary, input-output matrix parameters $\boldsymbol{\alpha}$, $\boldsymbol{\Omega}$, labor production share $\boldsymbol{\gamma}$, import-export share adjustment $\boldsymbol{\xi}$, and steady-state final use distribution $\boldsymbol{\theta}$ are the simple average of the following measures, respectively:

$$
\begin{aligned}
\boldsymbol{\Omega} =& [\omega_{ij}], \quad \omega_{ij} = \frac{\text{industry } i\text{'s use of industry } j\text{'s output}}{\sum_k \text{industry } i\text{'s use of industry } k\text{'s output}},\\
\boldsymbol{\alpha} =& (\alpha_i), \quad \alpha_i = \frac{\text{industry } i\text{'s total input expenditure}}{\text{industry } i\text{'s gross output}},\\
\boldsymbol{\gamma} =& (\gamma_i), \quad \gamma_i = \frac{\text{industry } i\text{'s labor compensation}}{\text{industry } i\text{'s gross output}},\\
\boldsymbol{\xi} =& (\xi_i), \quad \xi_i = \frac{\text{industry } i\text{'s gross sales}}{\text{final-use expenditure on } i\text{'s output} + \sum_j \text{industry } j\text{'s use of } i\text{'s output}},\\
\boldsymbol{\theta} =& (\theta_i), \quad \theta_i = \frac{\text{final-use expenditure on } i\text{'s output}}{\text{total final use expenditure}}.
\end{aligned}
$$

The input-output matrices are obtained for sector, summary, and detail aggregation levels to construct the (in)degree statistics. At the detail level, three industries (with BEA codes 4200ID,

814000, and S00201) show zero or negative total outputs and are removed from the matrix.

At the summary level, $\kappa = 13514$ minimizes the Frobenius norm (Euclidian norm of the vectorized matrix) of $\text{var}(\boldsymbol{\theta}_t) - \widehat{\text{var}}(\Delta\boldsymbol{\theta}_t^{data})$, where $\text{var}(\boldsymbol{\theta}_t)$ is the variance-covariance matrix of $\boldsymbol{\theta}_t$ as a Dirichlet random variable and $\widehat{\text{var}}(\Delta\boldsymbol{\theta}_t^{data})$ is obtained from data. The first-order difference is to take into account the trend in a few industries. Using $\boldsymbol{\theta}_t^{data}$ directly results in a much higher volatility on average and consequently a much smaller $\kappa = 2733$, which could give the micro shocks and signals in the model an unfair advantage. Using the L_2-norm instead of the Frobenius norm also gives a smaller value of $\kappa = 5435$.

The household's labor-supply condition implies that the data counterpart of quarterly $a_t = \ln A_t$ can be obtained by:

$$a_t = \ln h_t - \ln \frac{\text{total labor compensation}_t}{FU_t},$$

where h_t corresponds to total hours worked in the BLS. Then σ_η is calculated as the standard deviation of Δa_t.

C.4 Choosing Values of (T, N, σ_ϵ)

The moments to match are volatilities (standard deviations) $\sigma(\Delta s_{i,t}^{data})$ and $\sigma(\Delta u_{i,t}^{data})$, and serial correlation matrices with elements $\rho(\Delta s_{i,t}^{data}, \Delta s_{j,t+1}^{data})$ and $\rho(\Delta u_{i,t}^{data}, \Delta u_{j,t+1}^{data})$ for any pair of $1 \leq i, j \leq n$. Let $\boldsymbol{g}^{data}$ be the $(2n + 2n^2)$-dimensional vector containing the data moments. Similarly, $\boldsymbol{g}^{simul}(T, N, \sigma_\epsilon)$ contains the simulated moments. To obtain $\boldsymbol{g}^{simul}(T, N, \sigma_\epsilon)$, for each combination of (T, N, σ_ϵ), the model is simulated for 5000 periods with the first 500 periods discarded. To save simulation time, the multinomial distributions $\text{Multinomial}(N, \boldsymbol{\theta}_{t+\tau})$ for messages $\boldsymbol{m}_{t+\tau}^t$ are approximated by multivariable normal distributions. Specifically, $\boldsymbol{m}_{t+\tau}^t$ is drawn from $\mathcal{N}(\boldsymbol{\theta}_{t+\tau}, N^{-1}(\text{diag}(\boldsymbol{\theta}_{t+\tau}) - \boldsymbol{\theta}_{t+\tau}\boldsymbol{\theta}_{t+\tau}'))$. The goal is to find (T, N, σ_ϵ) that solves the following objective:

$$\min_{T,N\in\mathbb{N},\sigma_\epsilon>0} \left[\boldsymbol{g}^{simul}(T, N, \sigma_\epsilon) - \boldsymbol{g}^{data}\right]' W^{-1} \left[\boldsymbol{g}^{simul}(T, N, \sigma_\epsilon) - \boldsymbol{g}^{data}\right], \tag{12}$$

where the block-diagonal weight matrix W contains the variance of each target moment based on 1000 bootstrapped samples. Because of serial correlation in data, the stationary data series $\{\Delta\boldsymbol{s}_t^{data}, \Delta\boldsymbol{u}_t^{data}\}$ with sample size n_{data} are divided into non-overlapping blocks of length $(n_{data})^{1/3}$ and a block bootstrap sample is generated by sampling these blocks. Slight changes in the block length or using overlapping blocks do not alter the results much.

I calibrate four sets of (T, N, σ_ϵ)-values under two time-to-build specifications and two message set specifications, reported in the following table:

Values of (T, N, σ_ϵ) under two specifications of $\mathcal{M}_t$ in two versions of the model

Model	Message set $\mathcal{M}_t$	T	N/κ	$\sigma_\epsilon/\sigma_\eta$	Min. obj. value
Baseline	News	5	0.232	23.66	112182.80
	Learning	6	0.0448	225.94	112203.96
With just-in-time inputs	News	5	0.232	8.52	112521.66
	Learning	8	0.0305	109.21	112546.42

Table 3 in the main text shows the simulation results using parameter values obtained under the *news* specification in each model variant, *without* recalibration. The same statistics based on the other two sets of parameter values obtained under the *learning* specification are in Table D.1.

Note that both T and N are integers. Moreover, by design, T cannot be approximated by continuous real numbers. This prevents the application of asymptotic normality. Therefore, I present Figure C.1 to show the values of objective function (12) over a parameter grid. This grid is for illustration only. The actual parameters are picked using a genetic algorithm.

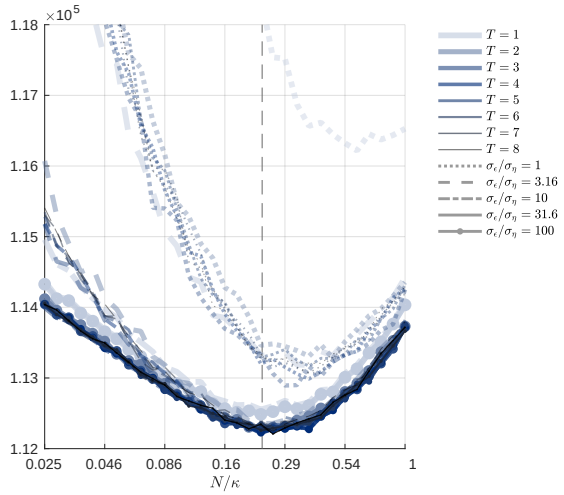
C.5 Capital Flows and the Interindustry Network Structure

The baseline model and its calibration are conservative in the interindustry connections' potential role in facilitating expectations-driven aggregate fluctuations. In addition to input-output linkages, another essential type of interindustry connection is the flow of capital purchases or leases across industries. As stated before, the absence of these flows weakens the model mechanism as they contribute to cross-sectional interindustry connections and over-time connections due to likely longer time to build capital. I use the BEA's 1997 capital flow table (after corrections) to examine how these additional connections may alter the network structure.

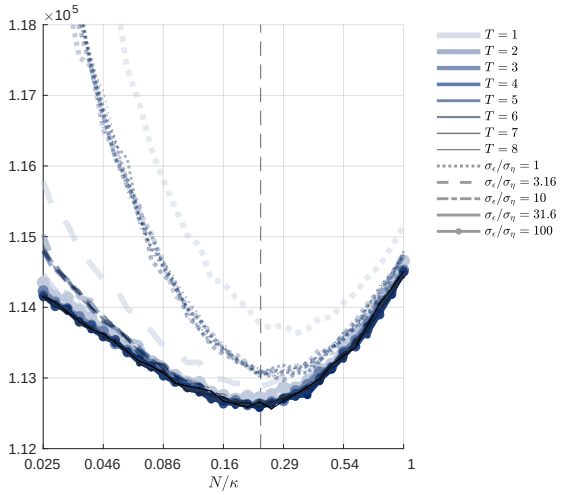
A capital flow table expands a portion of the final-use category "private fixed investment" (PFI, F02 in the use tables) to show the flow of new capitals by commodity to each private industry. The 1997 capital flow table is the most recent one available. It contains the flow of 180 commodities to 123 industries by NAICS codes. After aggregation, the table matches the "summary" aggregation level. Aggregation of the 1997 table is generally at a lower level than "summary", but it has a few more coarsely defined industries and commodities (retail and real estates). In these cases, the 1997 table's entry splits into corresponding summary-level industries proportional to these industries' 1997 output and into corresponding commodities proportional to the private investment expenditures on these commodities. The commodity-by-industry matrix is then turned into a square industry-by-industry capital flow matrix using the 1997 make table.

Calculating the capital-flow counterparts of $\boldsymbol{\alpha}$ and $\boldsymbol{\Omega}$ uses relevant 1997 variable values. As the capital flow table does not include the government industries, the following exercises consider two versions of capital flow and input-output matrices. The full version has all $n = 71$ private and government industries, where the capital flow table expands to include government investments as well (F06, F07, and F10 except the government consumption categories ending with a "C"). The *private only* version has $n = 66$ private industries only, where the input-output matrix is adjusted accordingly.

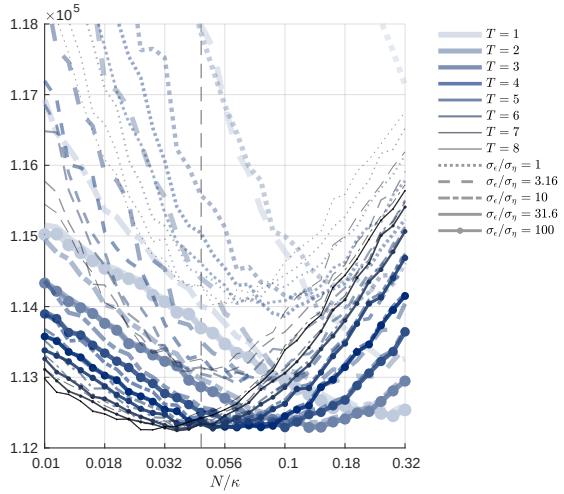
(a) Baseline, news



(b) JIT inputs, news



(c) Baseline, learning



(d) JIT inputs, learning

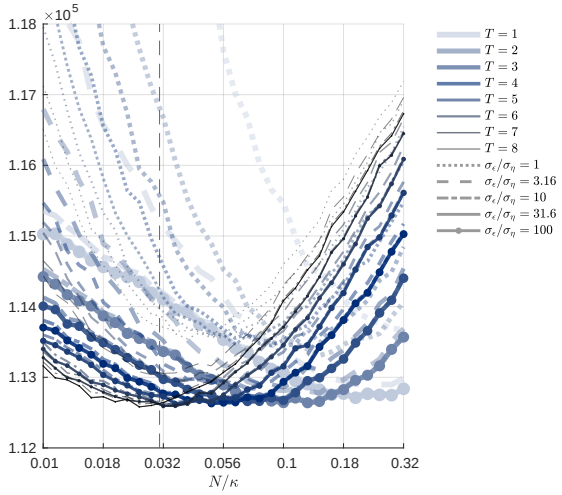


Figure C.1: Value of objective function (12) over a parameter subspace in each model specification.
Horizontal axes are N/κ, in linear $\log_{10}$-scale. Line widths (thick to thin) and shades (light to dark) correspond to T (1 to 8); line styles (dotted, dashed, broken, solid, marked) represent a few ascending values of $\sigma_\epsilon/\sigma_\eta$. Vertical dashed lines indicate the optimal value of $\sigma_\epsilon/\sigma_\eta$ in each case. Note that *learning* and *news* specifications are identical when $T = 1$.

Table C.1 and Figure C.2 use the cross-sectional distribution of weighted indegrees to illustrate how capital flows may strengthen the interindustry connections. Panel I of Table C.1, Figure C.2a, and Figure C.2b correspond to the baseline definition of weighted indegrees. When the adjacent matrix includes the interindustry capital flow, the resulting indegree distribution first-order stochastically dominates the original distribution with I-O linkages only. The cross-sectional mean and variance of indegrees increase. Whether we consider all producing industries or private industries only matters little for the statistics.

Panel II of Table C.1, Figure C.2c, and Figure C.2d consider the alternative definition of weighted indegrees, after adjusting for a subset of just-in-time inputs and/or capital. When both the I-O matrix and the capital flow matrix allow for just-in-time inputs simultaneously, we have a similar observation. Additionally, one may argue that capital goods and services require a longer time to build than intermediate inputs, especially if we have relatively low-frequency data. Therefore, I consider another time-to-build specification in which all inputs are just-in-time ones, and all capital flows take one period to complete from the supplier to the customer. The mean of first order indegrees remains higher than the original case, but the variance drops as the distribution becomes more concentrated. The magnitude drop in higher-order statistics is more pronounced. The intuition is that, compared to I-O linkages where long walks (chains of customer-supplier relationships) are common, it is relatively more difficult to find long walks for capital flows.

It is feasible to recalibrate the model taking into account interindustry flows of capital. However, as Figure C.3 illustrates, even if, in this case, we only use 1997 data, the model-implied steady-state Domar weight vector is not as close to the data counterpart as the baseline calibration strategy without capital flows. One reason is that capital flow tables report flows of *new* capital to *private* industries only. In the 1997 use table, the commodity category "scrap, used, and second-hand goods" (BEA commodity code "Used") alone accounts for nearly 5% (in absolute value) of total PFI, and the total government investment is above 20% of PFI. It is not entirely clear the best option to bridge the gap, in 1997 or in subsequent years where the capital flow tables are missing. Regardless, investments are part of the final use. In this sense, they are more similar to consumptions than they are to intermediate inputs. Therefore, the calibration strategy in the main text is preferred.

Table C.1: Indegree statistics of order τ for summary-level interindustry connections

Case	$\tau = 1$		$\tau = 2$		$\tau = 3$	
	$\overline{d^\tau}$	$\sqrt{V^{\text{in},\tau}}$	$\overline{d^\tau}$	$\sqrt{V^{\text{in},\tau}}$	$\overline{d^\tau}$	$\sqrt{V^{\text{in},\tau}}$
I. Baseline ($\boldsymbol{d}^{\text{in},\tau} = \boldsymbol{A}^\tau \mathbf{1}$)						
I-O	0.471	0.156	0.231	0.093	0.113	0.052
I-O + CF	0.594	0.167	0.359	0.110	0.219	0.071
I-O, private	0.482	0.154	0.239	0.093	0.119	0.053
I-O + CF, private	0.601	0.167	0.366	0.111	0.224	0.072
II. Allowing for just-in-time inputs ($\widetilde{\boldsymbol{d}^{\text{in},\tau}} = [(\boldsymbol{I} - \boldsymbol{A}_0)^{-1}\boldsymbol{A}_1]^\tau \mathbf{1}$)						
I-O	0.161	0.137	0.057	0.055	0.022	0.023
I-O + CF	0.296	0.142	0.139	0.073	0.068	0.036
I-O (all JIT) + CF (all TTB)	0.229	0.108	0.046	0.025	0.009	0.005
I-O, private	0.169	0.142	0.061	0.058	0.024	0.025
I-O + CF, private	0.305	0.148	0.145	0.077	0.072	0.038
I-O (all JIT) + CF (all TTB), private	0.228	0.105	0.046	0.025	0.009	0.005

- Panel I contains statistics of indegrees $\boldsymbol{d}^{\text{in},\tau} = \boldsymbol{A}^\tau \mathbf{1}$ as discussed in section 4.1, where the adjacent matrix $\boldsymbol{A}$ differs across cases. Panel II shows those of $\widetilde{\boldsymbol{d}^{\text{in},\tau}} = [(\boldsymbol{I} - \boldsymbol{A}_0)^{-1}\boldsymbol{A}_1]^\tau \mathbf{1}$ as in section 4.4, where $\boldsymbol{A} = \boldsymbol{A}_0 + \boldsymbol{A}_1$ and $\boldsymbol{A}_0$ represents the just-in-time part of the interindustry connections and $\boldsymbol{A}_1$ the part subject to one-period time to build.
- Case I-O indicates input-output linkages only, which coincides with the discussion in the main text. For consistency with the capital flow table, only the 1997 input-output matrix is used here, resulting in minor numerical differences in this case compared to Table 2's results at the summary level. Case I-O + CF means that the matrix $\boldsymbol{A}$ consists of both the input-output matrix and the capital flow matrix. In Panel II, in the case of I-O (all JIT) + CF (all TTB), *all* inputs are assumed to be just-in-time inputs and all flows of capital are subject to one-period time to build. Matrices in the cases of I-O and I-O + CF in Panel II are divided using the same rule as in the main text, such that the products of mining, utilities, construction, and durable-good manufacturing required one-period time to build.

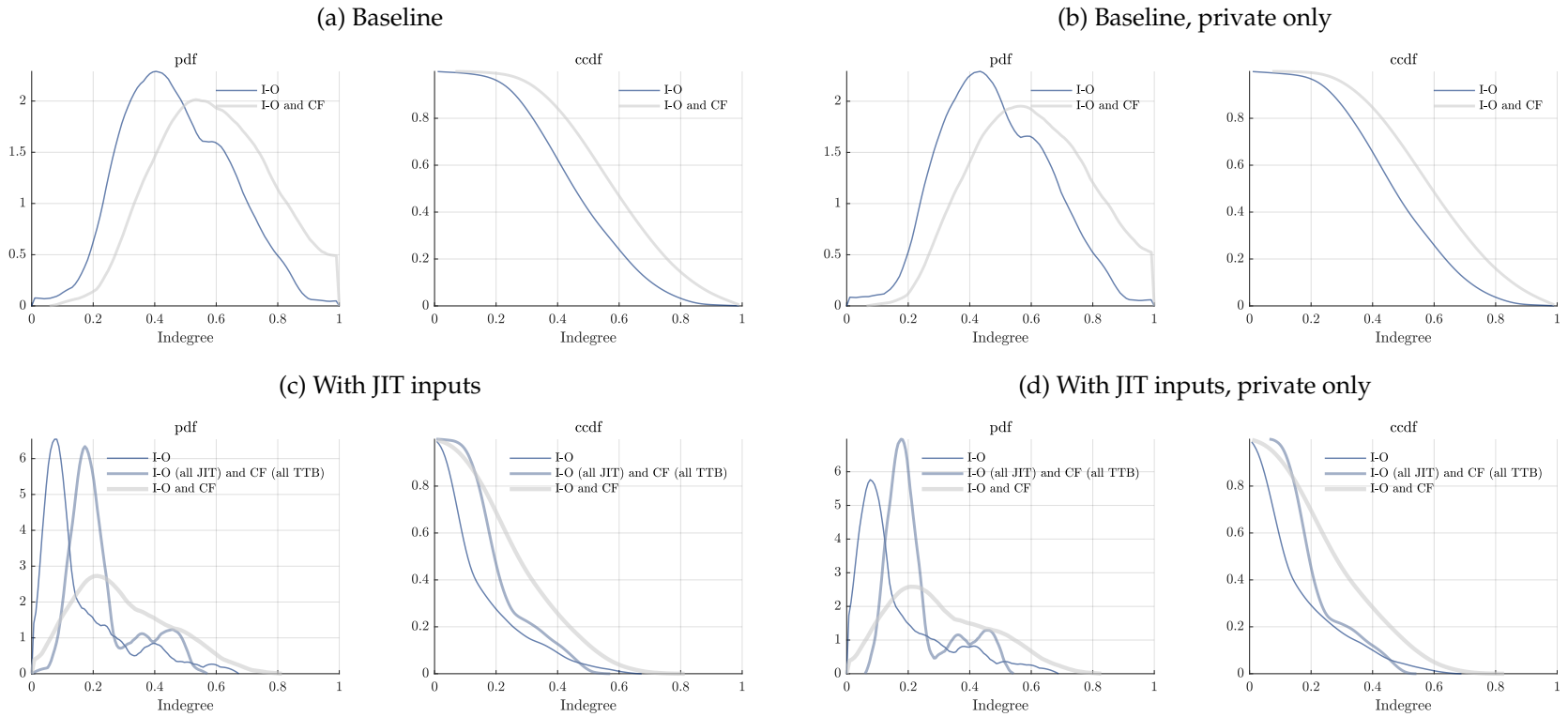


Figure C.2: Comparison of cross-sectional indegree distributions with and without capital flows.

Each figure plots the kernel density estimate of the density function (pdf) or the counter cumulative distribution function (ccdf=1-cdf) of the cross-sectional distribution of weighted indegrees of order $\tau = 1$. Figure C.2a and Figure C.2b correspond to the cases in Panel I of Table C.1; and Figure C.2c and Figure C.2d correspond to Panel II.

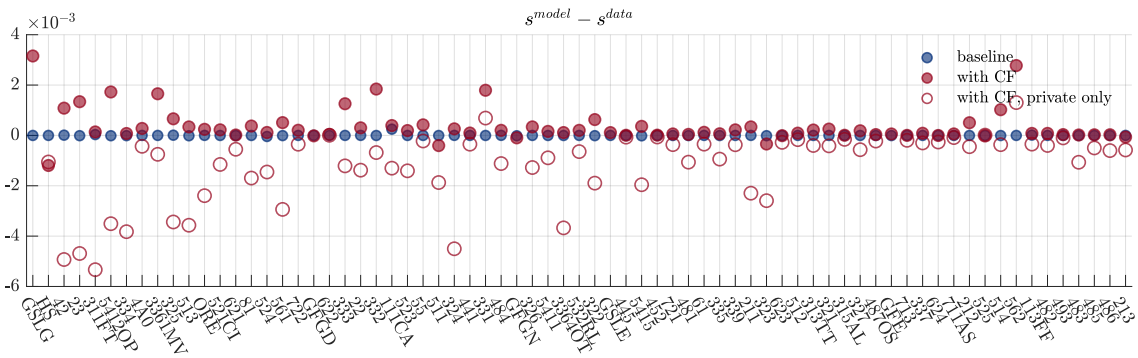


Figure C.3: Difference between model-implied steady-stead Domar weights and their data counterparts in 1997, with and without capital flows.

The horizontal axis shows the industry codes, in descending order of their 1997 gross output. Blue dots show the differences in Domar weights between model and data when the model calibration does not consider capital flows. The model is mapped to 1997 I-O matrix (with import-export adjustment), and the data counterparts of Domar weights are calculated as described in C.2. Red dots represent differences in s_i when the capital flow matrix is included as well, and the final-use shares are matched to the distribution of domestic final-use expenditures net all investments. The denominator for s_i^{data} is therefore changed to total domestic final use net all investments. Red circles are differences in s_i when only private industries are considered in the I-O and capital flow network, and the final-use shares are matched to the distribution of domestic and private final-use expenditures net private fixed investments. The denominator for s_i^{data} is total domestic and private final use less private fixed investments.

D Supplementary Figures and Tables

Table D.1: Table 3 and Table 4 using values of (T, N, σ_ϵ) calibrated based on the learning specification: simulated business-cycle moments

	Real GDP (Y)		Real Final Use (FU)			Hours (H)			Real Input (II)		
Specification of $\mathcal{M}_t$ [††]	σ_Y	ρ_Y	σ_{FU}/σ_Y	$\rho_{FU,Y}$	ρ_{FU}	σ_H/σ_Y	$\rho_{H,Y}$	ρ_H	σ_{II}/σ_Y	$\rho_{II,Y}$	ρ_{II}
Data	1.12	0.88	1.33	0.98	0.89	1.29	0.89	0.83	3.82	0.78	0.84
Baseline model ($T = 6,\ N/\kappa = 0.0448,\ \sigma_\epsilon/\sigma_\eta = 225.94$)											
Learning	0.80	0.69	0.87	0.92	0.77	1.84	1.00	0.69	0.88	0.92	0.78
News	0.80	0.69	0.87	0.92	0.77	1.84	1.00	0.69	0.87	0.92	0.77
Micro only, learning	0.18	−0.03	1.15	0.39	−0.17	1.62	0.92	−0.05	1.21	0.39	−0.04
Micro only, news	0.18	−0.03	1.12	0.36	−0.17	1.59	0.92	−0.06	1.14	0.37	−0.14
With just-in-time inputs[†] ($T = 8,\ N/\kappa = 0.0305,\ \sigma_\epsilon/\sigma_\eta = 109.21$)											
Learning	0.55	0.64	0.97	0.95	0.66	2.77	0.99	0.66	1.13	0.81	0.55
News	0.55	0.64	0.97	0.95	0.66	2.76	0.99	0.66	1.00	0.93	0.64
Micro only, learning	0.18	−0.04	0.98	0.53	−0.15	2.43	0.93	−0.06	2.08	0.31	0.14
Micro only, news	0.18	−0.04	0.97	0.54	−0.14	2.41	0.92	−0.06	1.21	0.45	0.01

All variables, both data series and model-simulated ones, are presented in their percentage deviation from the trends calculated by applying the Hodrick-Prescott (HP) filter with a quarterly smoothing parameter 1600 to the logarithms. Model generated statistics are based on the corresponding simulation of 5000 periods starting from steady state. The first 500 periods are discarded. For every variable X, σ_X means its standard deviation, $\rho_{X,Y}$ its correlation coefficient to real GDP (Y), and ρ_X its first order autocorrelation.

[†] All inputs are time-to-build inputs in the baseline model.

[‡] The message set specifications are the same as in the baseline model. The set of time-to-build inputs contains the products of 16 industries. The rest are all just-in-time inputs.

[††] Section 5.1 defines the learning and news specifications of the message set $\mathcal{M}_t$. Micro only means that macro uncertainty is shut down and all agents know that $A_t = A, \forall t$. The only source of uncertainty and fluctuations is micro fundamental shocks $\{\boldsymbol{\theta}_t\}$, and $\mathcal{M}_t$ contains micro signals only, depending on the learning or news specification.

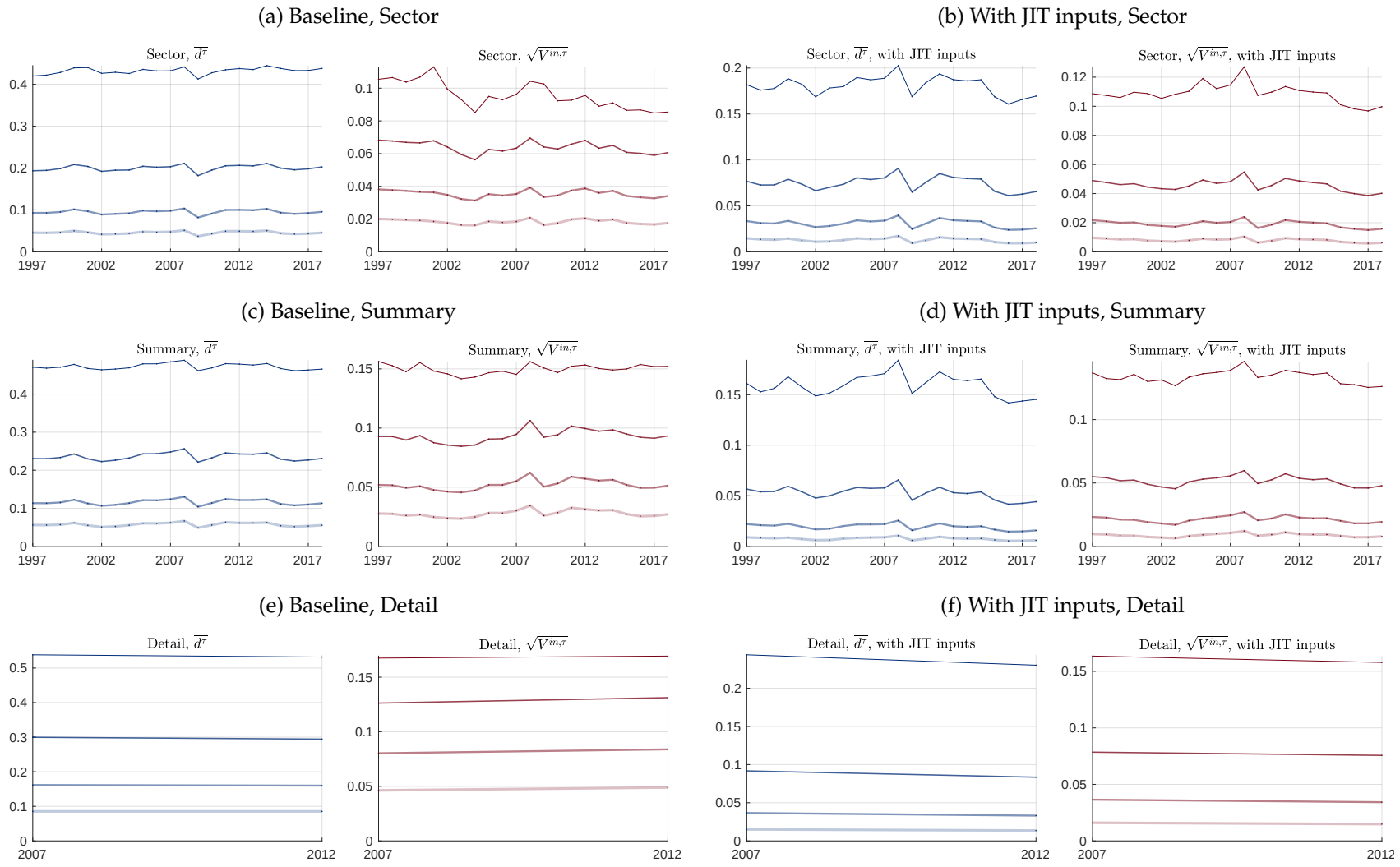


Figure D.1: Statistics of weighted idegrees $\overline{d^\tau}$ and $\sqrt{V^{\text{in},\tau}}$ in each sample year, for two specifications of time to build at three aggregation levels.

In each figure, from top to bottom, each line represents order $\tau = 1, 2, 3, 4$, respectively. Statistics at the "sector" or "summary" level are plotted annually, 1997–2018; those at the "detail" level are for 2007 and 2012. The variation in each statistics over time is observable, but not large enough to contradict the results of Table 2.

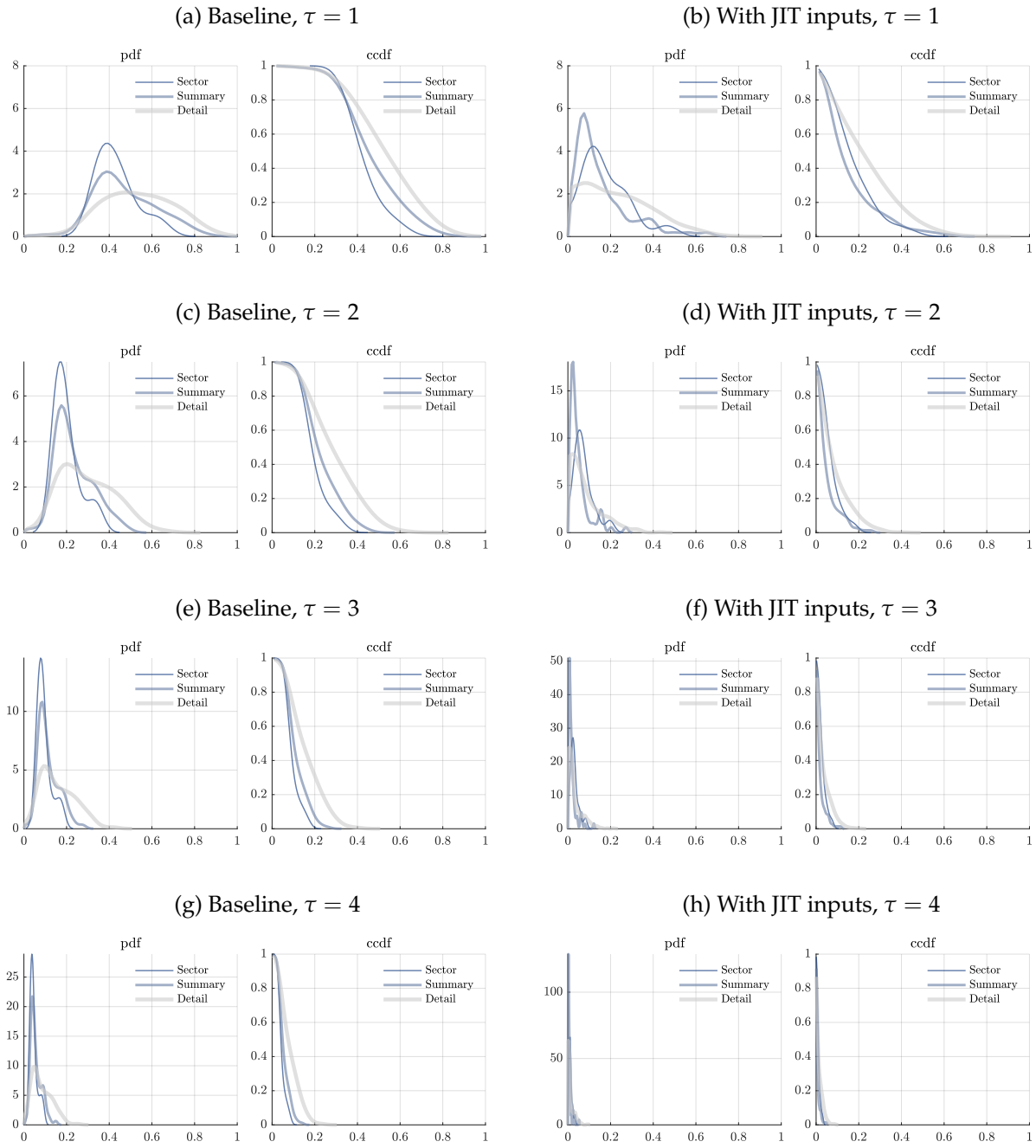


Figure D.2: Indegree distribution: cross-sectional density function (pdf) and counter cumulative distribution function (ccdf = 1-cdf) of $d_i^{\text{in},\tau}$, $\tau = 1, 2, 3, 4$, for two specifications of time to build at three aggregation levels.

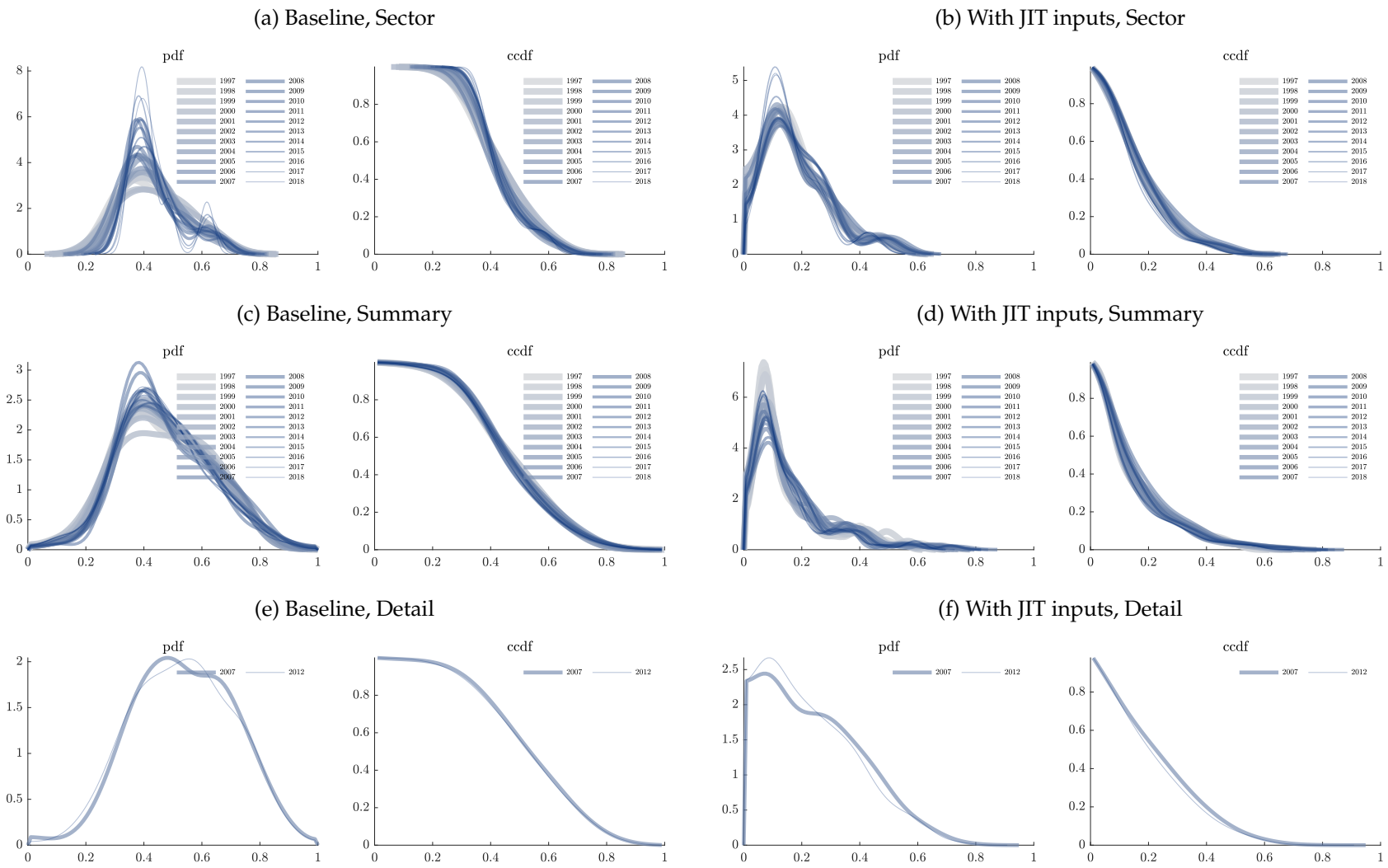
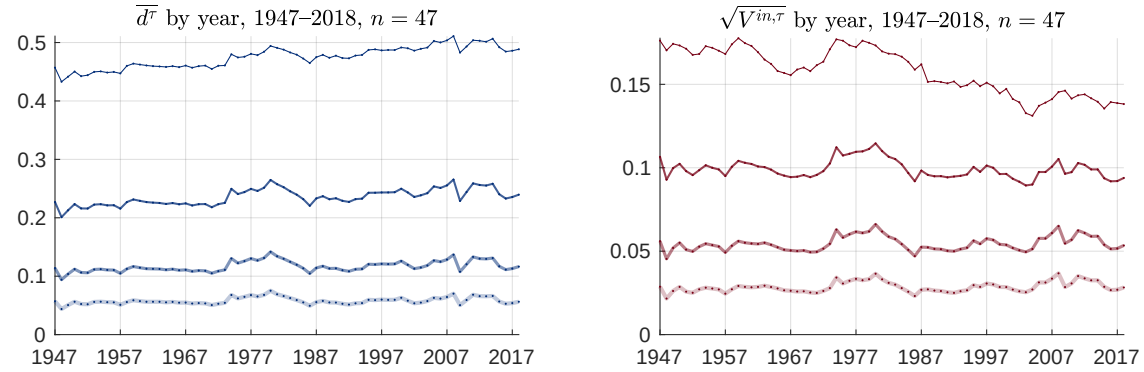


Figure D.3: Input share distribution in each sample year: cross-sectional density function (pdf) and counter cumulative distribution function (ccdf = 1-cdf) of $\alpha_i = d_i^{\text{in},1}$, for two specifications of time to build at three aggregation levels. Density functions at the "sector" or "summary" level are plotted annually, 1997–2018; those at the "detail" level are for 2007 and 2012.

(a) Baseline



(b) With JIT inputs

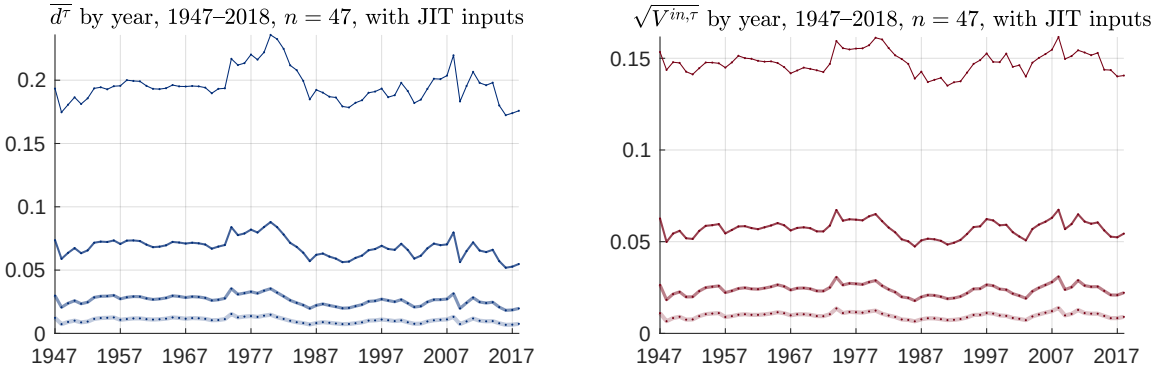


Figure D.4: Statistics of weighted indegrees $\overline{d^\tau}$ and $\sqrt{V^{\text{in},\tau}}$ from 1947 to 2018.

In each figure, from top to bottom, each line represents order $\tau = 1, 2, 3, 4$, respectively. Historical U.S. I-O tables are from the BEA industry account. More recent annual I-O tables are aggregated to match the $n = 47$ industries in the 1947–1962 annual tables. Without adjusting for time to build, these statistics reflect the structural changes in the input-output network. The most evident trend is that input shares ($\tau = 1$) become more concentrated at higher values over the years. After adjusting for time to build, the trend is not clear. An interesting observation is that, after such adjustments, both the mean degree and the dispersion in indegree peaked during 1970's to 1980's.

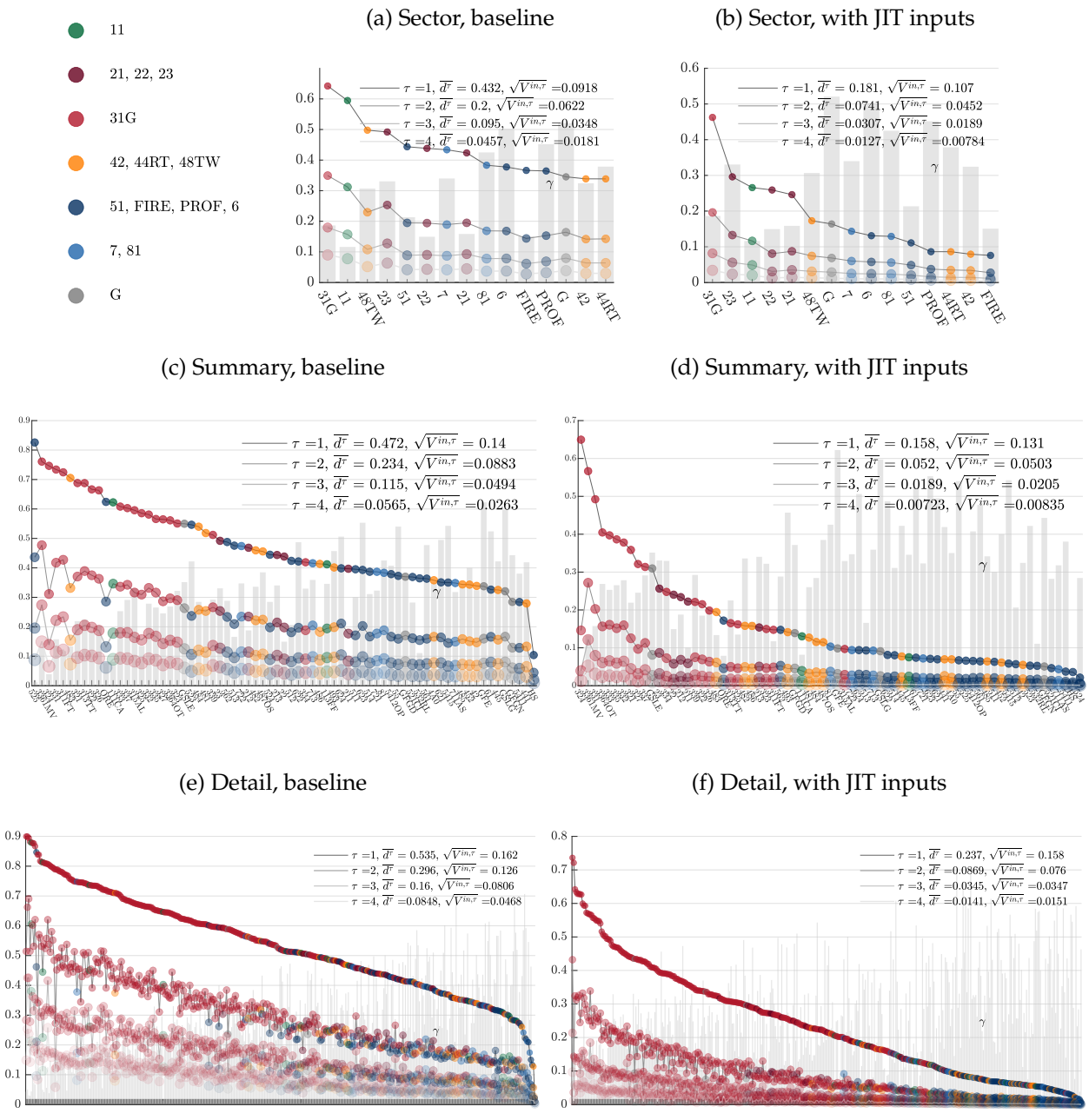


Figure D.5: Indegrees of orders $\tau = 1, 2, 3, 4$ (dark to light) by industry at each aggregation level. Horizontal axes show the I-O code of every industry i, in descending order of $\alpha_i = d_i^{\text{in},1}$ in the baseline case and in descending order of $\widetilde{d_i^{\text{in},1}}$ with JIT inputs. The hight of light gray bars in the background indicates average labor input share γ_i in the corresponding industry. Manufacturing industries (31G, red) tend to have higher indegrees of all orders at every aggregation level.

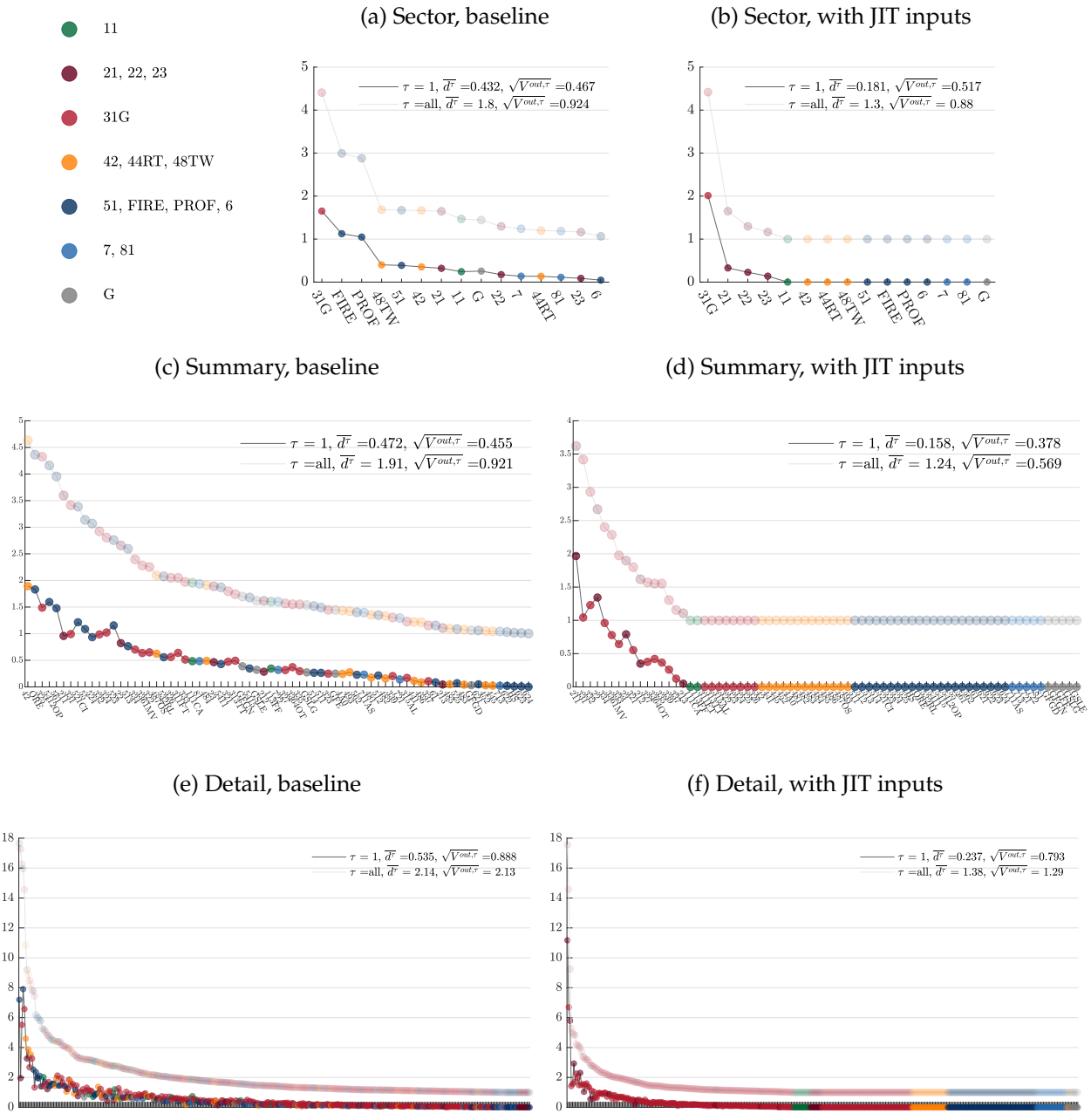
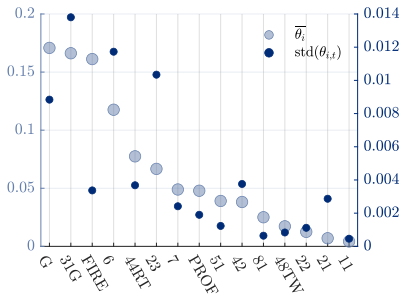
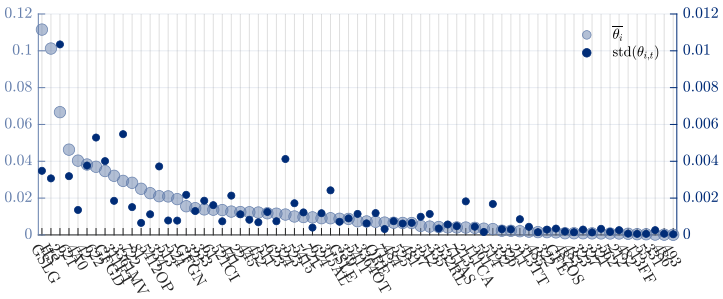


Figure D.6: Outdegrees $d_i^{\text{out},1}$ and $d_i^{\text{out, all}}$ by industry at each aggregation level. Horizontal axes show the I-O code of every industry i, in descending order of $d_i^{\text{out, all}}$ in the baseline case and in descending order of $\widetilde{d_i^{\text{out, all}}}$ with JIT inputs.

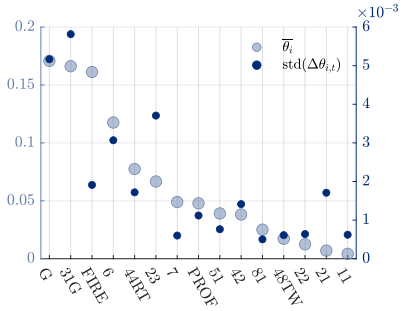
(a) Sector, θ_i and $\sigma(\theta_{i,t})$



(b) Summary, θ_i and $\sigma(\theta_{i,t})$



(c) Sector, θ_i and $\sigma(\Delta\theta_{i,t})$



(d) Summary, θ_i and $\sigma(\Delta\theta_{i,t})$

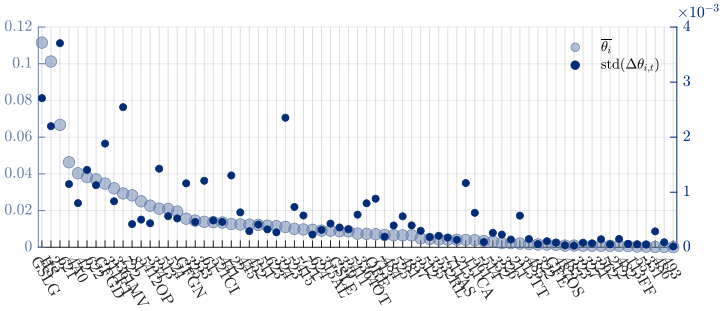
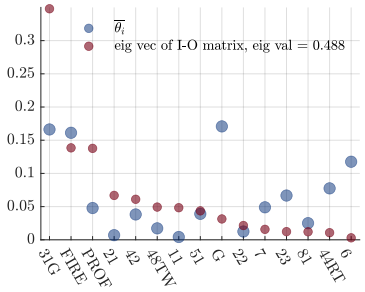
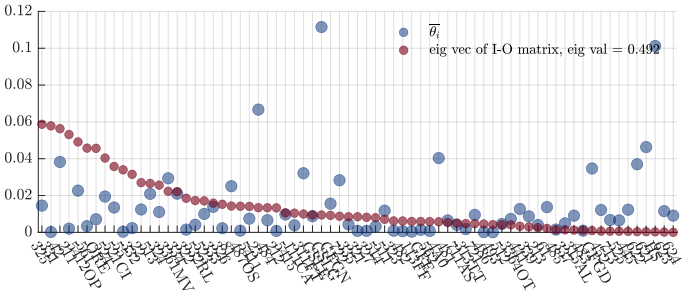


Figure D.7: Data average θ_i (left axis) versus volatility in $\theta_{i,t}$ (right axis) at two aggregation levels, in descending order of θ_i. If the final-use expenditure share vector indeed follows a Dirichlet distribution, then the standard deviation $\sigma(\theta_{i,t})$ should also be in descending order, as $\sigma(\theta_{i,t}) = \sqrt{\theta_i(1-\theta_i)/(\kappa+1)}$ and $\theta_{i,t} < 1/2$. Figure D.7a and Figure D.7b show that the assumption of a Dirichlet distribution is reasonable and a single parameter κ can capture much of the volatility in $\boldsymbol{\theta}_t$. In case of a secular trend, Figure D.7c and Figure D.7d show $\sigma(\Delta\theta_{i,t})$ instead, which conveys a similar message. The value of κ is picked based on $\text{var}(\Delta\boldsymbol{\theta}_t)$.

(a) Sector, $n = 15$



(b) Summary, $n = 71$



(c) Detail, $n = 402$

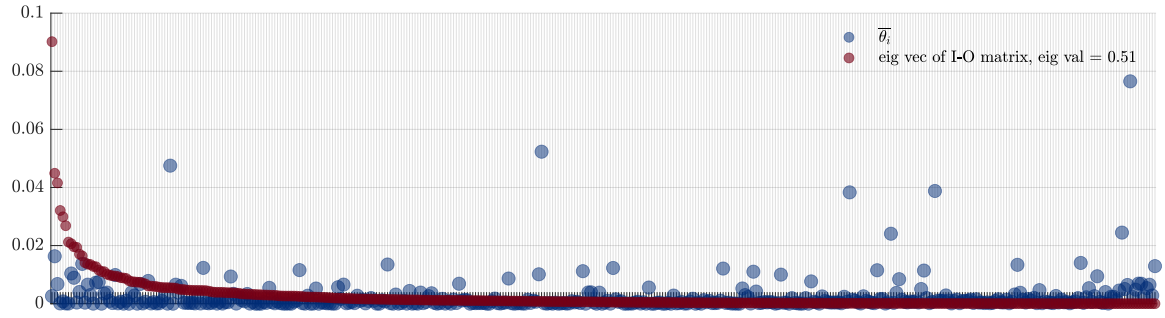


Figure D.8: Data actual $\boldsymbol{\theta}$ versus the eigenvector centrality measures of input-output network (the eigenvector associated with the largest eigenvalue of $\boldsymbol{\Omega}'\,\mathrm{diag}(\boldsymbol{\alpha})$, see section 4.3) at various aggregation level, in descending order of the eigenvector.