ECON 720: Advanced Macroeconomic Theory I

Introduction

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Fall 2026

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What is macroeconomics?

Hard to draw the line nowadays!

- Typical emphasis: an economy as a whole
  - over time: long-run trend (growth) and short-run fluctuations (business cycles)
  - cross-sectional: distribution over individuals (e.g., inequality)
- Typical theoretical method: dynamic general equilibrium
  - micro-foundation: explicit modeling of individual behavior and their interactions
  - aggregate outcome in all relevant markets in equilibrium

But exceptions are common.

This course vs. macroeconomic research

Typical components of a successful macro paper (besides a solid research question):

- **Empirical**: interesting patterns observed in data
- **Theoretical**: a model to match or explain the patterns
- **Quantitative**: calibration, validation, and practical use of the model

This course introduces a subset of **basic tools** often used in the **theoretical** and **quantitative** parts.

Course Topics

Main theme: **neoclassical growth model** (real: no monetary issues, smooth: no frictions or market imperfections)

- Starting point: the Solow model
- Deterministic neoclassical growth model in discrete time
  - Optimal growth and dynamic programming
  - Competitive equilibrium growth
  - Extensions of the basic model
- Stochastic neoclassic growth model in discrete time
- Continuous time methods

Typical workflow regarding a macroeconomic model

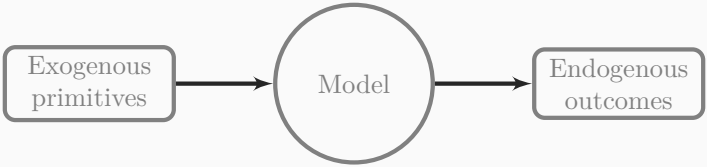
1. Set up the model
   - specify assumptions about stuff you don't want/need to explain
   - this is where you take a stand on the "boundary" of the model
2. Solve the model
   - obtain what you are interested in explaining
3. Calibrate and use the model
   - give the model reasonable parameter values and say something meaningful

Now we break this workflow step by step.

1. To set up a macroeconomic model, we need to specify...

- **Time** and **agents**
  - time: static or dynamic, discrete or continuous
  - agents: households and their demographics, firms, sometimes a government
- What each agent **has, does**, and **knows** at what time
  - households: preferences over commodities, endowments, decisions to maximize utility
  - firms: technologies, decisions to maximize profits
  - government: policies
- Market structures and **equilibrium** concept

2. To solve the model



Solving the model means to find the **endogenous** outcomes (e.g., equilibrium prices and allocations) given the **exogenous** primitives (e.g., preference, endowments, technology) under the **equilibrium** concept (e.g., a competitive equilibrium)

- In some special cases, mapping from primitives to endogenous outcomes is explicit (analytical solution)
- In most cases in modern macro, we go to the computer (numerical solution)

3. To calibrate and use the model

- Calibration: assign reasonable parameter values so the model mimics certain aspects of the real world (data)
  - Why matching the data? To ensure the numbers produced by model make some sense
- Possible uses of model: quantitative analysis and counterfactual experiments, e.g.,
  - explain interesting patterns observed in the data
  - quantify what cannot be directly observed in the data
  - evaluate hypothetical policies

A word of advice on the model workflow

- In this course:
  - 1 (setup) and 3 (use) are usually directly presented to you
  - 2 (solution) likely takes most of your time when learning the techniques
- However, in actual research, the **real modeling skill** is to find good 1 and 3
- So, try to pay attention to:
  - translation between economic stories and models
  - why an assumption is made; what each modeling choice buys and at what cost
  - possible uses of a model

Next...

- We'll see a simple and naive toy example that illustrates the workflow, especially the model-solving part.
- The model itself is not interesting and does not mean much.
- But solving and using more complex models often involves the same steps.

An illustrative example

Model setup

- Time: static, i.e., 1 period
- Agents: a representative household, a representative firm
  - a representative agent: think a unit-measure continuum of identical agents
- Household consumes, works, and rents out capital. Household preference over a consumption good is described by a pre-determined utility function $u(c) = \ln c$. Household is endowed with 1 unit of capital and 1 unit of labor, supplied inelastically.
- Firm rents capital k and hires labor l from corresponding markets to produce the consumption good. The production technology is described by a CRS Cobb-Douglas production function $f(k, l) = zk^{\alpha}l^{1-\alpha}$, where z is productivity (TFP). why?
- All markets (good, labor, capital) are perfectly competitive, i.e., household and firm are *price-takers* and make decisions based on prices only (instead of what others do)
- Equilibrium concept: a competitive equilibrium...

Equilibrium definition

Definition 1
A **competitive equilibrium** of this economy consists of ***prices*** including good price p^*, wage w^*, and rental rate r^*, and an ***allocation*** (i.e., quantities) including household's consumption demand c^*, firm's labor demand l^* and capital demand k^*, such that

1. (**individual optimization**) given prices (p^*, w^*, r^*), the allocation solves each agent's optimization problem, i.e., c^* solves the representative household's problem, and (k^*, l^*) solves the representative firm's problem;

2. (**market clearing**) all markets clear (demand = supply)

$$c = f(k^*, l^*), \qquad k^* = 1, \qquad l^* = 1.$$

Equilibrium definition is important, conceptually and practically!

Solving for the equilibrium

Taking stock:

- exogenous model primitives: functional forms of $u(c)$ and $f(k, l)$, parameters z and α, inelastic labor and capital supply
- endogenous equilibrium outcome: $p^*, w^*, r^*, c^*, l^*, k^*$ (to be solved)

To solve for an equilibrium typically involves:

- find out the equations that describe the equilibrium conditions, including
  - household optimization condition and firm optimization conditions
  - market clearing conditions (already known here)
- (analytically or numerically) solve the equations

Solving for the equilibrium: Step 1

Step 1. Write down the household's (constrained) optimization problem and solve it.

- **Household problem**: ***taking as given any prices*** (p, w, r), household faces the following constrained optimization problem:

$$\max_{c \geq 0} \ \ln c$$
$$\text{subject to } pc \leq w \times 1 + r \times 1.$$

  It means household chooses consumption level c to maximize utility subject to a budget constraint (expenditure$\leq$income).

- In this particular example, optimal consumption demand is trivial:

$$c = \frac{w + r}{p}.$$

  This describes consumption as a function of factor income $w + r$ and good price p.

Solving for the equilibrium: Step 2

Step 2. Write down the firm's optimization problem and solve it.

- **Firm problem**: ***taking as given any prices*** (p, w, r), firm faces the following profit optimization problem:

$$\max_{k\geq 0,\ l\geq 0} pzk^{\alpha}l^{1-\alpha} - wl - rk.$$

  It means firm chooses capital and labor inputs k and l to maximize profit.

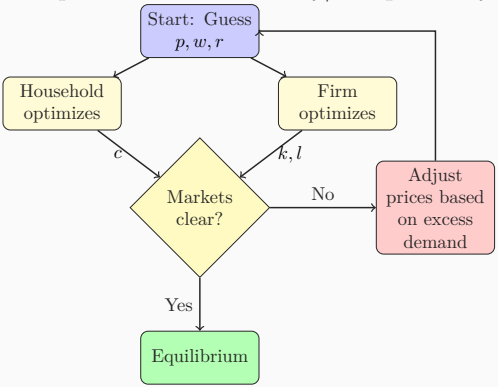
- Optimal demand for k and l (if interior) satisfy the first-order conditions (FOCs), i.e., MB = MC:

$$\partial k:\ \alpha pz\left(\frac{k}{l}\right)^{-(1-\alpha)} - r = 0, \qquad \partial l:\ (1-\alpha)pz\left(\frac{k}{l}\right)^{\alpha} - w = 0$$

  Note: we can at most solve for a capital-labor ratio $\frac{k}{l} = \frac{\alpha}{1-\alpha}\frac{w}{r}$ (scale indeterminacy). Maximized profit is 0, achieved at any pair of k and l satisfying this ratio.

Solving for the equilibrium: Step 3, numerically

Step 3. Check consistency/compatibility using the market clearing conditions.



CAUTION: This algorithm works conceptually, with a small issue: firm FOCs only determine the k/l ratio, not k and l individually. In practice, we use a market clearing condition ($k = 1$ or $l = 1$) to pin down the scale.

```
GUESS initial prices (p, w, r)
REPEAT:
    // Agents optimize independently
    c_demand = HouseholdProblem(p, w, r)
    (k_demand, l_demand) =
        FirmProblem(p, w, r)
    // Check if markets clear
    c_demand ?=
        Production(k_demand, l_demand)
    k_demand ?= 1
    l_demand ?= 1
    IF all markets clear:
        RETURN "Equilibrium"
    ELSE:
        UPDATE prices based on excess
            demand
```

Solving for the equilibrium: Step 3, analytically

Step 3. Check consistency/compatibility using the market clearing conditions $\Longrightarrow$ equilibrium conditions become a system of 6 equations and 6 unknowns:

$$\begin{aligned}
&\text{household optimization:} && c^* = \frac{w^* + r^*}{p^*} \\
&\text{firm optimization:} && \frac{r^*}{p^*} = \alpha z \left(\frac{k^*}{l^*}\right)^{-(1-\alpha)}, \quad \frac{w^*}{p^*} = (1-\alpha) z \left(\frac{k^*}{l^*}\right)^{\alpha} \\
&\text{market clearing:} && c^* = z k^{*\alpha} l^{*1-\alpha}, \quad k^* = 1, \quad l^* = 1
\end{aligned}$$

Prices only appear in relative terms, w^*/p^* and r^*/p^*. So we can normalize $p^* = 1$, i.e., we pick the consumption good as the *numeraire*. Solution is straightforward:

$$w^* = (1-\alpha)z, \qquad r^* = \alpha z, \qquad c^* = z, \qquad k^* = 1, \qquad l^* = 1.$$

A couple of things

Because we are looking for a competitive equilibrium:

- When analyzing household and firm problems, we *take prices as given*, where household and firm are price-takers and have no market power. This does NOT mean prices are exogenous!
- NEVER plug in the market clearing conditions (e.g., $k^* = 1$ and $l^* = 1$) before solving individual decision problems (e.g., writing down FOCs for firm problem)! Agents' decisions respond to prices only, not quantities on the other side of a market.

General equilibrium effect

- All three markets must clear at the same time and they are interdependent
- General equilibrium effect: "everything affects everything," or what happens in one market does not end there
  - **arguably the most important aspect of the "macro way of thinking"!**
- We can get a taste in this simple example: what if labor supply drops from 1 to 0.5?
  - intuitively, labor should be more expensive, so relative wage should go up
  - but that's not the full scope of the effect, relative rental rate changes too (keep $p^* = 1$)

$$w^* = (1-\alpha) z 2^{\alpha}, \qquad r^* = \frac{\alpha z}{2^{1-\alpha}}, \qquad c^* = \frac{z}{2^{1-\alpha}}.$$

Quantitative use of the model

We can ask the model some simple questions:

- Q: what happens to the economy if productivity increases by 10%?
  A: wage, rental/interest rate, and consumption increase by 10%

- Q: what happens to the economy if labor supply drops by 50%?
  A: wage increases by $2^{\alpha} - 1$, interest rate and consumption drop by $|2^{-(1-\alpha)} - 1|$

We can't get a quantitative answer to Q2 without knowing α!

Calibrating α

- What is a reasonable value of α? We ask the data.
- The idea is to find the α such that "something" produced by model matches data.
- In this case, we obtain from the model that

$$\frac{w^* h^*}{p^* y^*} = 1 - \alpha$$

where the left-hand side can be measured/observed from data, and the right-hand side is a function of the parameter. For example, we can use NIPA Table 2.1,

$$\frac{w^* h^*}{p^* y^*} = \text{labor share} = \frac{\text{total compensation of employees}}{\text{total personal income}} \approx \frac{2}{3} \implies \alpha = \frac{1}{3}$$

Market efficiency?

- The equilibrium allocation is efficient if it coincides with the planner's choice
- **Social planner's problem**: imagine a benevolent planner that chooses an allocation (quantities) to maximize household welfare using available technology and resources

$$\begin{aligned} &\max_{c,k,l\geq 0} \ln c \\ &\text{subject to (technology) } c \leq zk^{\alpha}l^{1-\alpha}, \\ &\qquad\qquad \text{(resource constraints) } k \leq 1,\ l \leq 1. \end{aligned}$$

- Efficient allocation is the same as the equilibrium outcome:

$$c^{**} = z, \qquad k^{**} = 1, \qquad l^{**} = 1.$$

This is not a coincidence.

About the planner

What is a social planner?

- NOT an agent! A hypothetical entity coordinating all activities in an economy.
- Benevolent: chooses **allocation** to maximize some welfare measure subject to resource and technological constraints

Why consider the planner's problem?

- First-best/efficient/Pareto optimal benchmark: markets never do better than planner
- Policy guidance: what's theoretically achievable in an economy
- Computational shortcut: whenever the first welfare theorem holds (you'll see it later), equilibrium allocation is Pareto optimal. Solving the planner's problem is usually much easier than solving for a competitive equilibrium directly.

GDP accounting: a quick reminder

Nominal GDP (aggregate output) in an economy over time period t (time index omitted):

$$\underbrace{Y = C + I + G + NX}_{\text{income-expenditure identity}} = \sum_{j \in \mathcal{J}} VA_j$$

three theoretically equivalent approaches:

- **income approach** (left): employees' compensation, interests, profits, etc.
- **expenditure approach** (middle): final uses, including private consumption, private investment, government purchases, net exports
- **value-added or product approach** (right): for any firm or industry $j \in \mathcal{J}$ (set of all firms or industries), value added is calculated as $VA_j = GO_j - II_j$, where GO_j is j's gross output, II_j is j's intermediate input uses of others' products, $II_j = \sum_{i \in \mathcal{J}} II_{ji}$.

Real GDP: nominal GDP with price changes removed using price indices (omitted)

GDP accounting and typical macro model ingredients

Why mentioning GDP accounting in a PhD course?

- representative household's budget constraints, e.g., $C + I = wL + rK + \Pi$
- aggregate or a representative firm's production function using *primary inputs* (such as capital and labor) but not *intermediate inputs*, e.g., $Y = F(K, L)$
  - in other words, $F(K, L)$ is a production function for *value added*
- final good market clearing condition or resource constraint, e.g., $C + I = F(K, L)$

Why a CRS production function?

Homogenous functions in general:

- A function $f : \mathbb{R}^n \to \mathbb{R}$ is homogeneous of degree m if

$$f(\lambda x) = \lambda^m f(x), \quad \forall \lambda > 0$$

- (**Euler's theorem**) If a differentiable function f is homogenous of degree m, then

$$mf(x) = \sum_{i=1}^{n} x_i f_i(x), \quad f_i := \partial f / \partial x_i.$$

  Proof: Differentiation wrt λ yields $m\lambda^{m-1} f(x) = \sum_{i=1}^{n} x_i f_i(\lambda x)$. Then set $\lambda = 1$. □

- If a differentiable function f is homogenous of degree m, then each of its partial derivatives f_i is homogenous of degree $m - 1$.
  Proof: Differentiation wrt x_i yields $\lambda f_i(\lambda x) = \lambda^m f_i(x)$, or $f_i(\lambda x) = \lambda^{m-1} f_i(x)$, $\forall i$. □

Why a CRS production function?

- A constant-returns-to-scale production function $F(K, L)$ is homogenous of degree 1

$$F(\lambda K, \lambda L) = \lambda F(K, L), \quad \forall \lambda > 0.$$

- A CRS production thus has the property $F(K, L) = F_K(K, L) \cdot K + F_L(K, L) \cdot L$. Under perfect competition, the firm's static profit maximization problem $\Pi := \max_{K,L>0} F(K, L) - rK - wL$ implies that $r = F_K(K, L), \quad w = F_L(K, L)$, so the firm's optimal profit is always $\Pi = 0$

- The optimal firm size is indeterminant, i.e., the firm is indifferent among all K and L satisfying the optimal capital-labor ratio, because

$$r = F_K(K, L) = F_K(K/L, 1), \quad w = F_L(K, L) = F_L(K/L, 1)$$

  With a common CRS technology, one large firm is equivalent to many small firms combined.

Readings

- AKMM: Ch.1